

# AP STATISTICS: TESTS OF SIGNIFICANCE

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## (1-Sample) z Test

STAT > TESTS > 1: Z-Test

Use this test to compare  $\bar{x}$  with  $\mu$  (if  $\sigma$  is known).

The question must provide  $\mu$ ,  $\sigma$ ,  $\bar{x}$ , and  $n$  or data from which these values can be computed.

$$z \text{ test statistic} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$\text{Confidence Interval (CI)} = \bar{x} \pm z^* (\sigma/\sqrt{n})$$

Assumptions:

- (1) SRS
- (2) Normal distribution
- (3) Independence

## 1-Proportion z Test

STAT > TESTS > 5: 1-PropZTest

Use this test to compare  $\hat{p}$  with  $p$ .

The question must provide  $x$ ,  $n$ , and  $p$  or data from which these values can be computed. (Sometimes, you must assume a value for  $p$ , such as 0.5 when you flip a coin).

$$z \text{ test statistic} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$\text{CI} = \hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Assumptions:

- (1) SRS
- (2) Rules of thumb:
  - a.  $n \leq 0.1N$
  - b.  $np \geq 10$  and  $n(1-p) \geq 10$

## 2-Sample z Test

STAT > TESTS > 3: 2-SampZ-Test

Use this test to compare  $\bar{x}_1 - \bar{x}_2$  with  $\mu_1 - \mu_2$ .

The question must provide  $\sigma_1$ ,  $\sigma_2$ ,  $\mu_1$ ,  $\mu_2$ ,  $\bar{x}_1$ ,  $\bar{x}_2$ ,  $n_1$ , and  $n_2$  or data from which these values can be computed.

$$z \text{ test statistic} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$\text{CI} = [(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)] \pm z^* \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Assumptions:

- (1) SRS
- (2)  $n_1 > 5$  and  $n_2 > 5$
- (3) Similar sample size and distribution

## 2-Proportion z Test

STAT > TESTS > 6: 2-PropZTest

Use this test to compare  $\hat{p}_1 - \hat{p}_2$  with  $p_1 - p_2$ .

The question must provide  $x_1$ ,  $x_2$ ,  $n_1$ , and  $n_2$  or data from which these values can be computed.

$$\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2}$$

$$z \text{ test statistic} = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_c(1-\hat{p}_c)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\text{CI} = (\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Assumptions:

- (1) SRS
- (2) Rules of thumb:
  - a.  $n_1 \leq 0.1N_1$  and  $n_2 \leq 0.1N_2$
  - b.  $n_1\hat{p}_1 \geq 10$  and  $n_1(1-\hat{p}_1) \geq 10$
  - c.  $n_2\hat{p}_2 \geq 10$  and  $n_2(1-\hat{p}_2) \geq 10$

**(1-Sample) t Test**      **STAT > TESTS > 2: T-Test**

Use this test to compare  $\bar{x}$  with  $\mu$  (if  $\sigma$  is NOT known).

The question usually provides a set of data from which you must compute  $\bar{x}$  and  $n$ .

$$t \text{ test statistic} = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$CI = \bar{x} \pm t^* (s/\sqrt{n})$$

$$\text{Degrees of freedom (df)} = n - 1$$

Assumptions:

- (1) SRS
- (2) How large is  $n$ ?
  - a.  $n < 15$  (No outliers and no skewing)
  - b.  $15 \leq n \leq 30$  (No large outliers and no strong skewing)
  - c.  $n > 30$  (Good to go)

**$\chi^2$  Test**      **STAT > TESTS > C:  $\chi^2$ -Test**

Use this test to compare observed values with expected values.

The question usually involves an  $r \times c$  table. (If the totals are missing, then you must compute them yourself).

$$\chi^2 \text{ test statistic} = \sum \frac{(O - E)^2}{E}$$

$$E = (\text{row total} \times \text{column total}) / \text{table total}$$

Assumptions:

- (1) SRS
- (2) All expected counts  $> 1$   
< 20% of the expected counts can be  $< 5$

**2-Sample t Test**      **STAT > TESTS > 4: 2-SampTTest**

Use this test to decide if the difference between two samples (which represent two populations) is statistically significant.

The question must provide  $\bar{x}_1$ ,  $\bar{x}_2$ ,  $n_1$ , and  $n_2$  or data from which these values can be computed.

$$t \text{ test statistic} = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$CI = (\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$df \text{ (conservative estimate)} = (\text{smaller } n) - 1$$

A more accurate  $df$  can be obtained using **2-SampTTest**.

Assumptions:

- (1) SRS
- (2)  $n_1 > 5$  and  $n_2 > 5$

Similar sample size and distribution

**MISCELLANEOUS INFORMATION**

Normal distribution notation:  $N(\mu, \sigma/\sqrt{n})$

Binomial distribution notation:  $B(n, p)$

Empirical rule (normal distributions):

- $\mu \pm \sigma$       ~ 68% of data
- $\mu \pm 2\sigma$       ~ 95% of data
- $\mu \pm 3\sigma$       ~ 99.7% of data

Binomial check: NITC (fixed # of trials, independent, two, constant probability)

$$\text{Binomial } \mu = np \qquad \text{Binomial } \sigma = \sqrt{np(1-p)}$$

$$z^* \text{ for 90\% CI } (\alpha = 0.10) = 1.644853626$$

$$z^* \text{ for 95\% CI } (\alpha = 0.05) = 1.959963986$$

$$z^* \text{ for 99\% CI } (\alpha = 0.01) = 2.575829303$$

Note that whereas  $z^*$  remains constant as  $n \rightarrow \infty$ ,  $t^*$  decreases due to the increased degrees of freedom.

Standard error (SE) = denominator of test stat formula  
OR the part of the CI formula after the critical value

Margin of error = second part of CI formula (after  $\pm$ )