

## AP Calculus AB - Summer Assignment

You have chosen to participate in a College Level Mathematics course. Please realize that taking this class is a major commitment and requires a lot of work. Honesty and integrity are crucial at this level of education and are demanded here. Any infractions will be dealt with according to the school Integrity Policy.

The summer assignment for AP Calculus AB is a review of functions, limits and graphing calculator procedures. This work will serve as an example of the type of assignments and effort that will be expected of you throughout the next school year. The assignment will not be collected. Instead, a **30 point quiz** based on the summer assignment material will be given on the very **first day** of school.

Solutions to the odd numbered textbook problems are available for free on the web at [www.CalcChat.com](http://www.CalcChat.com) or [www.hotmath.com](http://www.hotmath.com). Choose the textbook Calculus 9<sup>th</sup> edition by Larson. When using CalcChat, use the red circular button with the four yellow arrows to scroll down through the problem. To use HotMath, you need the password dccs462. When you choose a problem, if it does not appear to be the correct problem, adjust the number up or down until you get the correct problem.

A TI-89 Graphing Calculator is **REQUIRED** for this class. Prior to class in September, make sure you are familiar with a TI-89 and are able to use it on problems dealing with the following topics:

- Graphing functions
- Finding intercepts
- Finding relative maximums and minimums
- Finding the intersections of graphs
- Using the TABLE feature
- Using the SOLVE feature to solve equations

## Assignment:

All questions should be completed without a graphing calculator unless otherwise specified.

### Section P.1 - Graphs and Models: pages 8-9

In Exercises 19–28, find any intercepts.

19.  $y = 2x - 5$

20.  $y = 4x^2 + 3$

21.  $y = x^2 + x - 2$

22.  $y^2 = x^3 - 4x$

23.  $y = x\sqrt{16 - x^2}$

24.  $y = (x - 1)\sqrt{x^2 + 1}$

25.  $y = \frac{2 - \sqrt{x}}{5x}$

26.  $y = \frac{x^2 + 3x}{(3x + 1)^2}$

27.  $x^2y - x^2 + 4y = 0$

28.  $y = 2x - \sqrt{x^2 + 1}$

In Exercises 63–70, find the points of intersection of the graphs of the equations.

63.  $x + y = 8$

64.  $3x - 2y = -4$

$4x - y = 7$

$4x + 2y = -10$

65.  $x^2 + y = 6$

66.  $x = 3 - y^2$

$x + y = 4$

$y = x - 1$



In Exercises 71–74, use a graphing utility to find the points of intersection of the graphs. Check your results analytically.

71.  $y = x^3 - 2x^2 + x - 1$

72.  $y = x^4 - 2x^2 + 1$

$y = -x^2 + 3x - 1$


$y = 1 - x^2$

73.  $y = \sqrt{x + 6}$

74.  $y = -|2x - 3| + 6$


$y = \sqrt{-x^2 - 4x}$

$y = 6 - x$

-  **76. Modeling Data** The table shows the numbers of cellular phone subscribers (in millions) in the United States for selected years. (Source: Cellular Telecommunications and Internet Association)

Year	1990	1993	1996	1999	2002	2005
Number	5	16	44	86	141	208

- (a) Use the regression capabilities of a graphing utility to find a mathematical model of the form  $y = at^2 + bt + c$  for the data. In the model,  $y$  represents the number of subscribers and  $t$  represents the year, with  $t = 0$  corresponding to 1990.
- (b) Use a graphing utility to plot the data and graph the model. Compare the data with the model.
- (c) Use the model to predict the number of cellular phone subscribers in the United States in the year 2015.

-  **78. Copper Wire** The resistance  $y$  in ohms of 1000 feet of solid copper wire at  $77^\circ\text{F}$  can be approximated by the model

$$y = \frac{10,770}{x^2} - 0.37, \quad 5 \leq x \leq 100$$

### WRITING ABOUT CONCEPTS

In Exercises 79 and 80, write an equation whose graph has the indicated property. (There may be more than one correct answer.)

79. The graph has intercepts at  $x = -4$ ,  $x = 3$ , and  $x = 8$ .
80. The graph has intercepts at  $x = -\frac{3}{2}$ ,  $x = 4$ , and  $x = \frac{5}{2}$ .
81. (a) Prove that if a graph is symmetric with respect to the  $x$ -axis and to the  $y$ -axis, then it is symmetric with respect to the origin. Give an example to show that the converse is not true.
- (b) Prove that if a graph is symmetric with respect to one axis and to the origin, then it is symmetric with respect to the other axis.

**CAPSTONE**

82. Match the equation or equations with the given characteristic.

(i)  $y = 3x^3 - 3x$    (ii)  $y = (x + 3)^2$    (iii)  $y = 3x - 3$

(iv)  $y = \sqrt[3]{x}$    (v)  $y = 3x^2 + 3$    (vi)  $y = \sqrt{x + 3}$

(a) Symmetric with respect to the  $y$ -axis

(b) Three  $x$ -intercepts

(c) Symmetric with respect to the  $x$ -axis

(d)  $(-2, 1)$  is a point on the graph

(e) Symmetric with respect to the origin

(f) Graph passes through the origin

**Section P.3 - Functions and Their Graphs: pages 27-30**

In Exercises 1 and 2, use the graphs of  $f$  and  $g$  to answer the following.

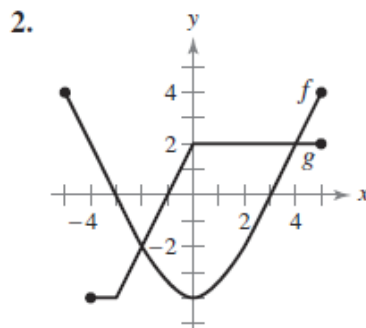
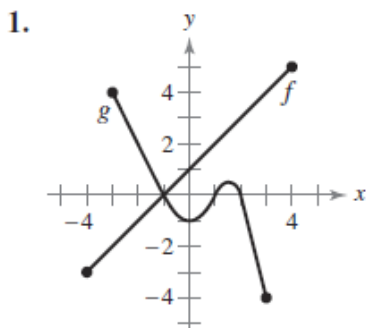
(a) Identify the domains and ranges of  $f$  and  $g$ .

(b) Identify  $f(-2)$  and  $g(3)$ .

(c) For what value(s) of  $x$  is  $f(x) = g(x)$ ?

(d) Estimate the solution(s) of  $f(x) = 2$ .

(e) Estimate the solutions of  $g(x) = 0$ .



In Exercises 3–12, evaluate (if possible) the function at the given value(s) of the independent variable. Simplify the results.

3.  $f(x) = 7x - 4$

(a)  $f(0)$

(b)  $f(-3)$

(c)  $f(b)$

(d)  $f(x - 1)$

5.  $g(x) = 5 - x^2$

(a)  $g(0)$

(b)  $g(\sqrt{5})$

(c)  $g(-2)$

(d)  $g(t - 1)$

7.  $f(x) = \cos 2x$

(a)  $f(0)$

(b)  $f(-\pi/4)$

(c)  $f(\pi/3)$

9.  $f(x) = x^3$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x}$$

11.  $f(x) = \frac{1}{\sqrt{x-1}}$

$$\frac{f(x) - f(2)}{x - 2}$$

4.  $f(x) = \sqrt{x+5}$

(a)  $f(-4)$

(b)  $f(11)$

(c)  $f(-8)$

(d)  $f(x + \Delta x)$

6.  $g(x) = x^2(x - 4)$

(a)  $g(4)$

(b)  $g(\frac{3}{2})$

(c)  $g(c)$

(d)  $g(t + 4)$

8.  $f(x) = \sin x$

(a)  $f(\pi)$

(b)  $f(5\pi/4)$

(c)  $f(2\pi/3)$

10.  $f(x) = 3x - 1$

$$\frac{f(x) - f(1)}{x - 1}$$

12.  $f(x) = x^3 - x$

$$\frac{f(x) - f(1)}{x - 1}$$

In Exercises 13–20, find the domain and range of the function.

13.  $f(x) = 4x^2$

14.  $g(x) = x^2 - 5$

15.  $g(x) = \sqrt{6x}$

16.  $h(x) = -\sqrt{x+3}$

17.  $f(t) = \sec \frac{\pi t}{4}$

18.  $h(t) = \cot t$

In Exercises 21–26, find the domain of the function.

21.  $f(x) = \sqrt{x} + \sqrt{1-x}$

22.  $f(x) = \sqrt{x^2 - 3x + 2}$

23.  $g(x) = \frac{2}{1 - \cos x}$

24.  $h(x) = \frac{1}{\sin x - \frac{1}{2}}$

25.  $f(x) = \frac{1}{|x+3|}$

26.  $g(x) = \frac{1}{|x^2 - 4|}$

In Exercises 27–30, evaluate the function as indicated. Determine its domain and range.

$$27. f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$$

(a)  $f(-1)$     (b)  $f(0)$     (c)  $f(2)$     (d)  $f(t^2 + 1)$

$$28. f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$$

(a)  $f(-2)$     (b)  $f(0)$     (c)  $f(1)$     (d)  $f(s^2 + 2)$

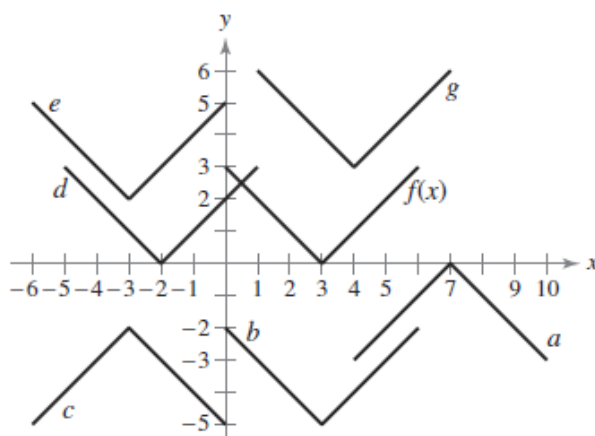
$$29. f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \geq 1 \end{cases}$$

(a)  $f(-3)$     (b)  $f(1)$     (c)  $f(3)$     (d)  $f(b^2 + 1)$

$$30. f(x) = \begin{cases} \sqrt{x + 4}, & x \leq 5 \\ (x - 5)^2, & x > 5 \end{cases}$$

(a)  $f(-3)$     (b)  $f(0)$     (c)  $f(5)$     (d)  $f(10)$

In Exercises 49–54, use the graph of  $y = f(x)$  to match the function with its graph.



49.  $y = f(x + 5)$

50.  $y = f(x) - 5$

51.  $y = -f(-x) - 2$

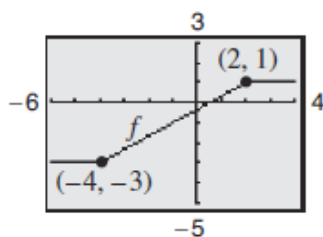
52.  $y = -f(x - 4)$

53.  $y = f(x + 6) + 2$

54.  $y = f(x - 1) + 3$

56. Use the graph of  $f$  shown in the figure to sketch the graph of each function. To print an enlarged copy of the graph, go to the website [www.mathgraphs.com](http://www.mathgraphs.com).

- (a)  $f(x - 4)$     (b)  $f(x + 2)$   
 (c)  $f(x) + 4$     (d)  $f(x) - 1$   
 (e)  $2f(x)$         (f)  $\frac{1}{2}f(x)$



60. Given  $f(x) = \sin x$  and  $g(x) = \pi x$ , evaluate each expression.

- (a)  $f(g(2))$         (b)  $f\left(g\left(\frac{1}{2}\right)\right)$     (c)  $g(f(0))$   
 (d)  $g\left(f\left(\frac{\pi}{4}\right)\right)$     (e)  $f(g(x))$         (f)  $g(f(x))$

In Exercises 61–64, find the composite functions  $(f \circ g)$  and  $(g \circ f)$ . What is the domain of each composite function? Are the two composite functions equal?

61.  $f(x) = x^2$   
 $g(x) = \sqrt{x}$

62.  $f(x) = x^2 - 1$   
 $g(x) = \cos x$

63.  $f(x) = \frac{3}{x}$

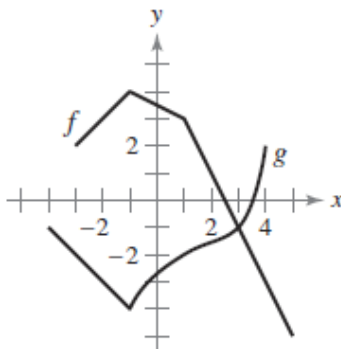
$g(x) = x^2 - 1$

64.  $f(x) = \frac{1}{x}$

$g(x) = \sqrt{x + 2}$

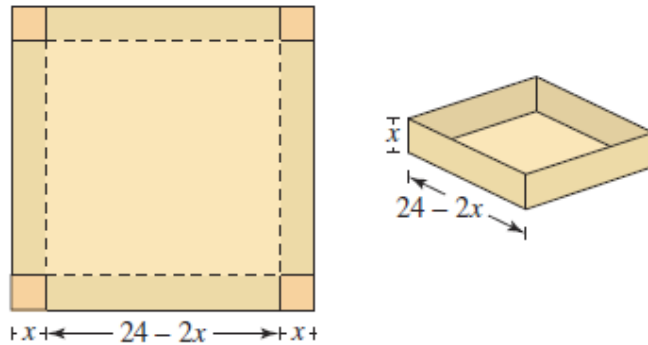
65. Use the graphs of  $f$  and  $g$  to evaluate each expression. If the result is undefined, explain why.

- (a)  $(f \circ g)(3)$         (b)  $g(f(2))$   
 (c)  $g(f(5))$         (d)  $(f \circ g)(-3)$   
 (e)  $(g \circ f)(-1)$     (f)  $f(g(-1))$





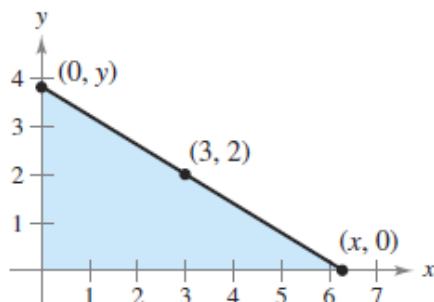
97. **Volume** An open box of maximum volume is to be made from a square piece of material 24 centimeters on a side by cutting equal squares from the corners and turning up the sides (see figure).



- (a) Write the volume  $V$  as a function of  $x$ , the length of the corner squares. What is the domain of the function?
- (b) Use a graphing utility to graph the volume function and approximate the dimensions of the box that yield a maximum volume.
- (c) Use the *table* feature of a graphing utility to verify your answer in part (b). (The first two rows of the table are shown.)

Height, $x$	Length and Width	Volume, $V$
1	$24 - 2(1)$	$1[24 - 2(1)]^2 = 484$
2	$24 - 2(2)$	$2[24 - 2(2)]^2 = 800$

98. **Length** A right triangle is formed in the first quadrant by the  $x$ - and  $y$ -axes and a line through the point  $(3, 2)$  (see figure). Write the length  $L$  of the hypotenuse as a function of  $x$ .





**Section P.R - Review Exercises: pages 37-38**

20. *Think About It* For what value of  $k$  does the graph of  $y = kx^3$  pass through the point?

- (a)  $(1, 4)$    (b)  $(-2, 1)$    (c)  $(0, 0)$    (d)  $(-1, -1)$

**In Exercises 23 and 24, use the concept of slope to find  $t$  such that the three points are collinear.**

23.  $(-8, 5), (0, t), (2, -1)$       24.  $(-3, 3), (t, -1), (8, 6)$

37. Evaluate (if possible) the function  $f(x) = 1/x$  at the specified values of the independent variable, and simplify the results.

- (a)  $f(0)$       (b)  $\frac{f(1 + \Delta x) - f(1)}{\Delta x}$

38. Evaluate (if possible) the function at each value of the independent variable.

$$f(x) = \begin{cases} x^2 + 2, & x < 0 \\ |x - 2|, & x \geq 0 \end{cases}$$

- (a)  $f(-4)$    (b)  $f(0)$    (c)  $f(1)$

39. Find the domain and range of each function.

(a)  $y = \sqrt{36 - x^2}$    (b)  $y = \frac{7}{2x - 10}$    (c)  $y = \begin{cases} x^2, & x < 0 \\ 2 - x, & x \geq 0 \end{cases}$

40. Given  $f(x) = 1 - x^2$  and  $g(x) = 2x + 1$ , evaluate each expression.

- (a)  $f(x) - g(x)$    (b)  $f(x)g(x)$    (c)  $g(f(x))$



45. *Area* A wire 24 inches long is to be cut into four pieces to form a rectangle whose shortest side has a length of  $x$ .
- Write the area  $A$  of the rectangle as a function of  $x$ .
  - Determine the domain of the function and use a graphing utility to graph the function over that domain.
  - Use the graph of the function to approximate the maximum area of the rectangle. Make a conjecture about the dimensions that yield a maximum area.

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**Section P.5 - Problem Solving: pages 39-40**

5. A rancher plans to fence a rectangular pasture adjacent to a river. The rancher has 100 meters of fencing, and no fencing is needed along the river (see figure).
- Write the area  $A$  of the pasture as a function of  $x$ , the length of the side parallel to the river. What is the domain of  $A$ ?
  - Graph the area function  $A(x)$  and estimate the dimensions that yield the maximum amount of area for the pasture.
  - Find the dimensions that yield the maximum amount of area for the pasture by completing the square.

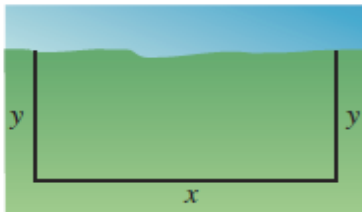


Figure for 5

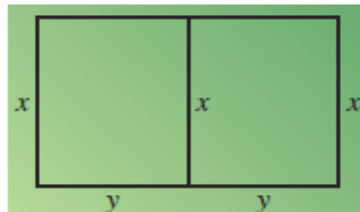
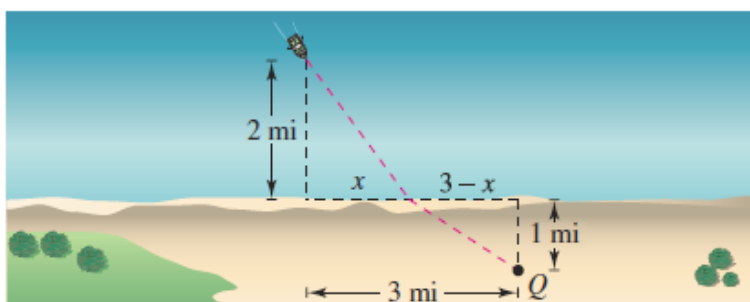


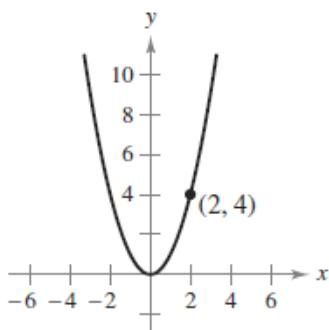
Figure for 6

6. A rancher has 300 feet of fencing to enclose two adjacent pastures.
- Write the total area  $A$  of the two pastures as a function of  $x$  (see figure). What is the domain of  $A$ ?
  - Graph the area function and estimate the dimensions that yield the maximum amount of area for the pastures.
  - Find the dimensions that yield the maximum amount of area for the pastures by completing the square.

7. You are in a boat 2 miles from the nearest point on the coast. You are to go to a point  $Q$  located 3 miles down the coast and 1 mile inland (see figure). You can row at 2 miles per hour and walk at 4 miles per hour. Write the total time  $T$  of the trip as a function of  $x$ .



8. You drive to the beach at a rate of 120 kilometers per hour. On the return trip, you drive at a rate of 60 kilometers per hour. What is your average speed for the entire trip? Explain your reasoning.
9. One of the fundamental themes of calculus is to find the slope of the tangent line to a curve at a point. To see how this can be done, consider the point  $(2, 4)$  on the graph of  $f(x) = x^2$  (see figure).



- Find the slope of the line joining  $(2, 4)$  and  $(3, 9)$ . Is the slope of the tangent line at  $(2, 4)$  greater than or less than this number?
- Find the slope of the line joining  $(2, 4)$  and  $(1, 1)$ . Is the slope of the tangent line at  $(2, 4)$  greater than or less than this number?
- Find the slope of the line joining  $(2, 4)$  and  $(2.1, 4.41)$ . Is the slope of the tangent line at  $(2, 4)$  greater than or less than this number?
- Find the slope of the line joining  $(2, 4)$  and  $(2 + h, f(2 + h))$  in terms of the nonzero number  $h$ . Verify that  $h = 1$ ,  $-1$ , and  $0.1$  yield the solutions to parts (a)(c) above.
- What is the slope of the tangent line at  $(2, 4)$ ? Explain how you arrived at your answer.

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**Section 1.2 - Finding Limits Graphically and Numerically: pages 54-58**

In Exercises 1–8, complete the table and use the result to estimate the limit. Use a graphing utility to graph the function to confirm your result.

6.  $\lim_{x \rightarrow 4} \frac{[x/(x+1)] - (4/5)}{x-4}$

$x$	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$						

7.  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$

$x$	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

8.  $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x}$

$x$	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

In Exercises 9–14, create a table of values for the function and use the result to estimate the limit. Use a graphing utility to graph the function to confirm your result.

9.  $\lim_{x \rightarrow 1} \frac{x-2}{x^2+x-6}$

10.  $\lim_{x \rightarrow -3} \frac{x+3}{x^2+7x+12}$

11.  $\lim_{x \rightarrow 1} \frac{x^4-1}{x^6-1}$

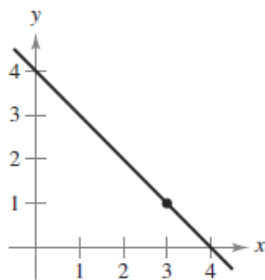
12.  $\lim_{x \rightarrow -2} \frac{x^3+8}{x+2}$

13.  $\lim_{x \rightarrow 0} \frac{\sin 2x}{x}$

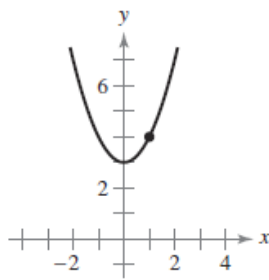
14.  $\lim_{x \rightarrow 0} \frac{\tan x}{\tan 2x}$

In Exercises 15–24, use the graph to find the limit (if it exists).  
If the limit does not exist, explain why.

15.  $\lim_{x \rightarrow 3} (4 - x)$

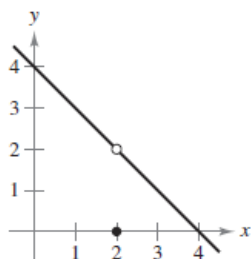


16.  $\lim_{x \rightarrow 1} (x^2 + 3)$



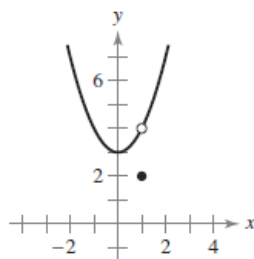
17.  $\lim_{x \rightarrow 2} f(x)$

$$f(x) = \begin{cases} 4 - x, & x \neq 2 \\ 0, & x = 2 \end{cases}$$

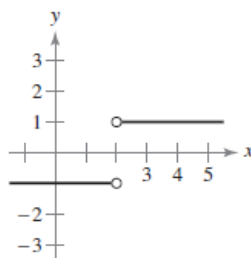


18.  $\lim_{x \rightarrow 1} f(x)$

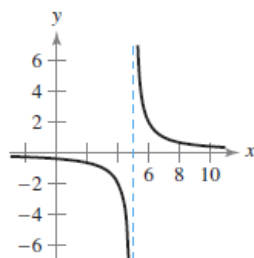
$$f(x) = \begin{cases} x^2 + 3, & x \neq 1 \\ 2, & x = 1 \end{cases}$$



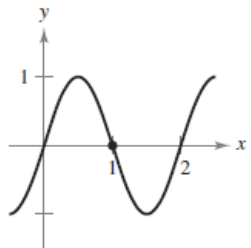
19.  $\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$



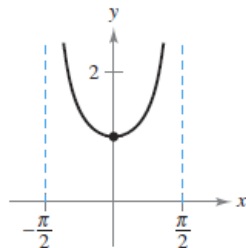
20.  $\lim_{x \rightarrow 5} \frac{2}{x - 5}$



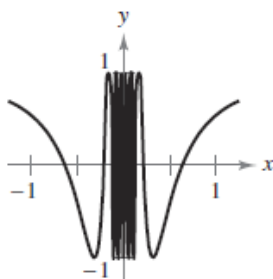
21.  $\lim_{x \rightarrow 1} \sin \pi x$



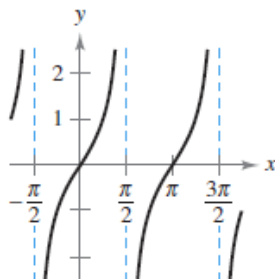
22.  $\lim_{x \rightarrow 0} \sec x$



23.  $\lim_{x \rightarrow 0} \cos \frac{1}{x}$

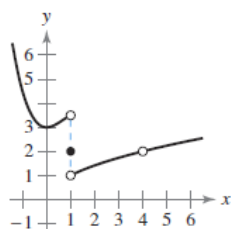


24.  $\lim_{x \rightarrow \pi/2} \tan x$

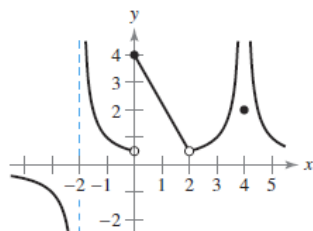


In Exercises 25 and 26, use the graph of the function  $f$  to decide whether the value of the given quantity exists. If it does, find it. If not, explain why.

25. (a)  $f(1)$   
 (b)  $\lim_{x \rightarrow 1} f(x)$   
 (c)  $f(4)$   
 (d)  $\lim_{x \rightarrow 4} f(x)$



26. (a)  $f(-2)$   
 (b)  $\lim_{x \rightarrow -2} f(x)$   
 (c)  $f(0)$   
 (d)  $\lim_{x \rightarrow 0} f(x)$   
 (e)  $f(2)$   
 (f)  $\lim_{x \rightarrow 2} f(x)$   
 (g)  $f(4)$   
 (h)  $\lim_{x \rightarrow 4} f(x)$



**Section 1.3 - Evaluating Limits Analytically: pages 67-69**

In Exercises 5–22, find the limit.

5.  $\lim_{x \rightarrow 2} x^3$                       6.  $\lim_{x \rightarrow -2} x^4$   
 15.  $\lim_{x \rightarrow -4} (x + 3)^2$               16.  $\lim_{x \rightarrow 0} (2x - 1)^3$   
 17.  $\lim_{x \rightarrow 2} \frac{1}{x}$                           18.  $\lim_{x \rightarrow -3} \frac{2}{x + 2}$

In Exercises 23–26, find the limits.

23.  $f(x) = 5 - x$ ,  $g(x) = x^3$

(a)  $\lim_{x \rightarrow 1} f(x)$       (b)  $\lim_{x \rightarrow 4} g(x)$       (c)  $\lim_{x \rightarrow 1} g(f(x))$

24.  $f(x) = x + 7$ ,  $g(x) = x^2$

(a)  $\lim_{x \rightarrow -3} f(x)$       (b)  $\lim_{x \rightarrow 4} g(x)$       (c)  $\lim_{x \rightarrow -3} g(f(x))$

25.  $f(x) = 4 - x^2$ ,  $g(x) = \sqrt{x + 1}$

(a)  $\lim_{x \rightarrow 1} f(x)$       (b)  $\lim_{x \rightarrow 3} g(x)$       (c)  $\lim_{x \rightarrow 1} g(f(x))$

26.  $f(x) = 2x^2 - 3x + 1$ ,  $g(x) = \sqrt[3]{x + 6}$

(a)  $\lim_{x \rightarrow 4} f(x)$       (b)  $\lim_{x \rightarrow 21} g(x)$       (c)  $\lim_{x \rightarrow 4} g(f(x))$

In Exercises 27–36, find the limit of the trigonometric function.

27.  $\lim_{x \rightarrow \pi/2} \sin x$

28.  $\lim_{x \rightarrow \pi} \tan x$

29.  $\lim_{x \rightarrow 1} \cos \frac{\pi x}{3}$

30.  $\lim_{x \rightarrow 2} \sin \frac{\pi x}{2}$

31.  $\lim_{x \rightarrow 0} \sec 2x$

32.  $\lim_{x \rightarrow \pi} \cos 3x$

33.  $\lim_{x \rightarrow 5\pi/6} \sin x$

34.  $\lim_{x \rightarrow 5\pi/3} \cos x$

35.  $\lim_{x \rightarrow 3} \tan\left(\frac{\pi x}{4}\right)$

36.  $\lim_{x \rightarrow 7} \sec\left(\frac{\pi x}{6}\right)$

In Exercises 37–40, use the information to evaluate the limits.

37.  $\lim_{x \rightarrow c} f(x) = 3$

38.  $\lim_{x \rightarrow c} f(x) = \frac{3}{2}$

$\lim_{x \rightarrow c} g(x) = 2$

$\lim_{x \rightarrow c} g(x) = \frac{1}{2}$

(a)  $\lim_{x \rightarrow c} [5g(x)]$

(a)  $\lim_{x \rightarrow c} [4f(x)]$

(b)  $\lim_{x \rightarrow c} [f(x) + g(x)]$

(b)  $\lim_{x \rightarrow c} [f(x) + g(x)]$

(c)  $\lim_{x \rightarrow c} [f(x)g(x)]$

(c)  $\lim_{x \rightarrow c} [f(x)g(x)]$

(d)  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$

(d)  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$

39.  $\lim_{x \rightarrow c} f(x) = 4$

40.  $\lim_{x \rightarrow c} f(x) = 27$

(a)  $\lim_{x \rightarrow c} [f(x)]^3$

(a)  $\lim_{x \rightarrow c} \sqrt[3]{f(x)}$

(b)  $\lim_{x \rightarrow c} \sqrt{f(x)}$

(b)  $\lim_{x \rightarrow c} \frac{f(x)}{18}$

(c)  $\lim_{x \rightarrow c} [3f(x)]$

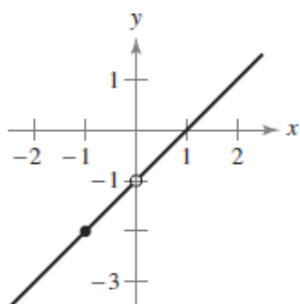
(c)  $\lim_{x \rightarrow c} [f(x)]^2$

(d)  $\lim_{x \rightarrow c} [f(x)]^{3/2}$

(d)  $\lim_{x \rightarrow c} [f(x)]^{2/3}$

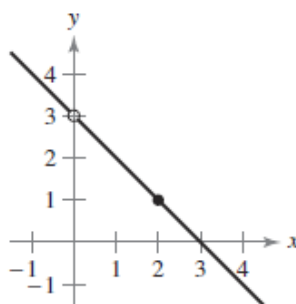
In Exercises 41–44, use the graph to determine the limit visually (if it exists). Write a simpler function that agrees with the given function at all but one point.

41.  $g(x) = \frac{x^2 - x}{x}$



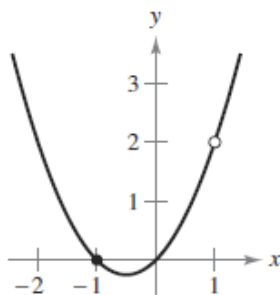
- (a)  $\lim_{x \rightarrow 0} g(x)$   
 (b)  $\lim_{x \rightarrow -1} g(x)$

42.  $h(x) = \frac{-x^2 + 3x}{x}$



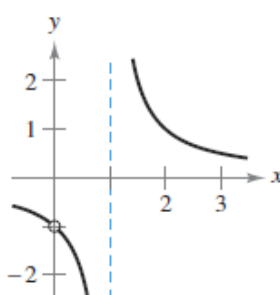
- (a)  $\lim_{x \rightarrow 2} h(x)$   
 (b)  $\lim_{x \rightarrow 0} h(x)$

43.  $g(x) = \frac{x^3 - x}{x - 1}$



- (a)  $\lim_{x \rightarrow 1} g(x)$   
 (b)  $\lim_{x \rightarrow -1} g(x)$

44.  $f(x) = \frac{x}{x^2 - x}$



- (a)  $\lim_{x \rightarrow 1} f(x)$   
 (b)  $\lim_{x \rightarrow 0} f(x)$

In Exercises 85–88, find  $\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$ .

85.  $f(x) = 3x - 2$

86.  $f(x) = \sqrt{x}$

87.  $f(x) = \frac{1}{x + 3}$

88.  $f(x) = x^2 - 4x$