Summary Exercises

The exercises that follow cover five of the major topic areas in AP Statistics, and the questions include, in the topic areas covered, the great majority of the facts, knowledge, and skills that are required on the exam. You are encouraged to use your class notes and your textbook when completing these exercises. Be sure, also, to refer to the Top Tips for the topic area you're working on. (The Top Tips start on page 1.)

Solutions to these summary exercises are provided in the Student's Solutions Manual.

By working through these exercises, and by checking your answers in the Student's Solutions Manual, you can greatly enhance your knowledge and ability, and significantly improve your performance on the AP Statistics exam.

CORRELATION AND LINEAR REGRESSION

1. A student at a large high school conducts a study using a random sample of 20 male seniors. The student records the height (in inches) and the length of the right foot (in millimeters) of each student selected, and uses a computer to fit a least squares regression line to the data. Part of the computer output is shown below.

```
Dependent variable: Foot Length

Predictor    Coef      SE Coef     T        P
Constant     148.17    42.88       3.46     0.003
Height       1.8849    0.5964      3.16     0.005

S = 6.85709     R-sq = 35.7%     R-sq (adj) = 32.1%
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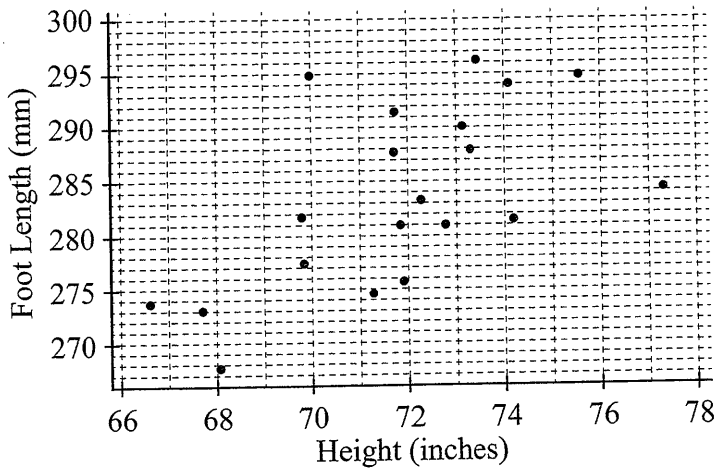
(a) What is the value of the correlation coefficient for foot length and height?

(b) Interpret the value of the correlation coefficient that you calculated in part (a).

(c) What is the equation of the least squares regression line for predicting foot length from height?

(d) A scatterplot of the results is shown below. Draw the least squares regression line on the scatterplot.



(e) Interpret the value of the slope of the least squares regression line in this context.

(f) Does the intercept of the least squares regression line have a meaningful interpretation in this context? If so, provide this interpretation. If not, explain why not.

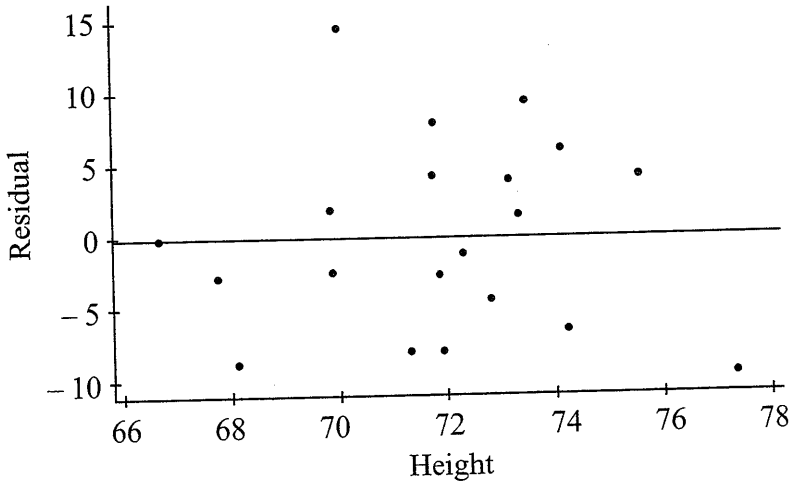
(g) What does the least squares regression line predict for the foot length of a student whose height is 73 inches?

(h) Would it be appropriate to use the fitted regression equation to predict the foot length for a student whose height is 62 inches? Explain your answer.

(i) One of the students in the study had a height of 71.8 inches and a foot length of 291.5 millimeters. Calculate the residual for that student.

(j) Interpret the value of the residual you calculated in part (i).

(k) A residual plot for this data set is shown below.



Is a line an appropriate model for the heights and foot lengths of these students? Explain how you reach your conclusion.

(l) State the value of r^2, and interpret this value in the context of this study.

(m) Note that $s = 6.85709$ in the computer output. Interpret this value in the context of this study.

(n) Identify and interpret the standard error of the slope.

(o) Suppose that a system of shoe sizes is formulated, where

$$\text{shoe size} = \frac{(\text{foot length}) - 285}{5}.$$

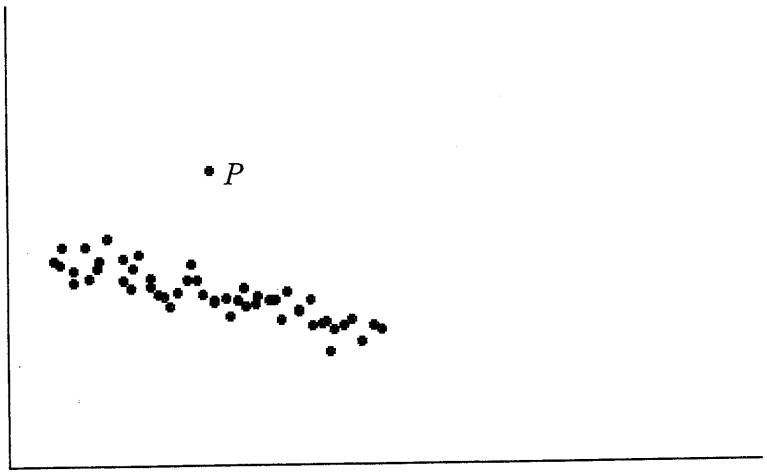
If, using this formula, the foot length is replaced by the shoe size for each student (with no rounding), what would be the resulting value of the correlation coefficient for shoe size and height?

2. Suppose that the heights and foot lengths of 15 high school senior girls are measured. The heights are found to have mean 63.8 inches and standard deviation 4.2 inches and the foot lengths are found to have mean 223.3 millimeters and standard deviation 12.7 millimeters. Additionally, the correlation between height and foot length for these girls is found to be 0.548. Let x = height in inches and y = foot length in millimeters.

   (a) Calculate the slope of the least squares regression line of y on x.

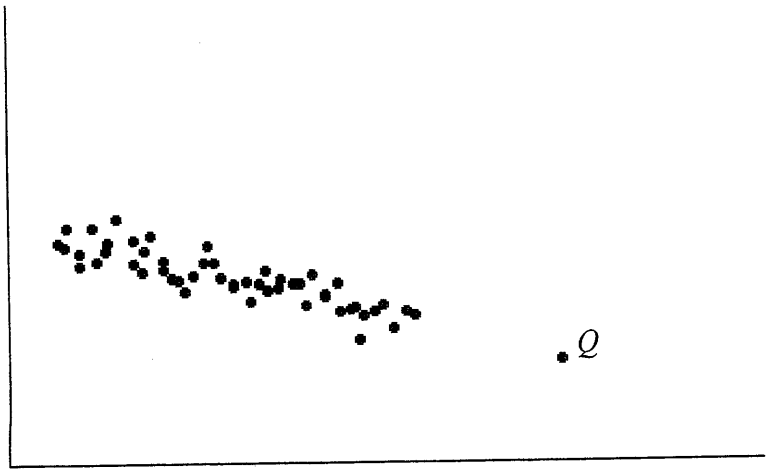
   (b) Calculate the y-intercept of the least squares regression line of y on x.

3. (a) Consider the data set represented by the scatterplot shown below.



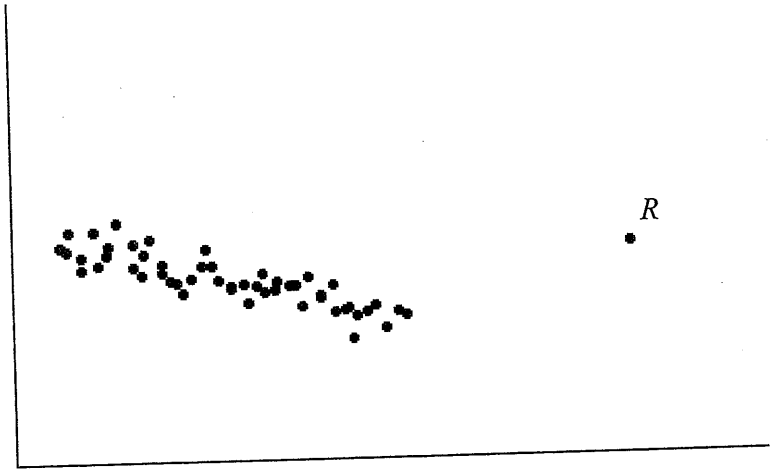
   Would removal of the point P result in a large change in the least squares regression line? Explain your answer.

   (b) Consider the data set represented by the scatterplot shown below.



Would removal of the point Q result in a large change in the least squares regression line? Explain your answer.

(c) Consider the data set represented by the scatterplot shown below.



Would removal of the point R result in a large change in the least squares regression line? Explain your answer.

EXPERIMENTAL DESIGN

1. Random Assignment

An experiment is to be designed to compare two drugs, Drug A and Drug B, both designed to reduce blood pressure. (Drug A is a new formulation. Drug B is a formulation that has been used for some years.) You have been provided with 40 volunteers, all of whom suffer from high blood pressure. Twenty of the volunteers will receive Drug A, and the other 20 will receive Drug B. The reduction in blood pressure will be measured for each participant.

(a) Explain how the 40 volunteers might be randomly assigned to the two groups.

(b) Why would the volunteers be assigned *randomly* to the two groups, rather than, for example, allowing each volunteer (independently of the other volunteers) to choose which group he/she would be in?

(c) Suppose the blood pressures of the volunteers who are given Drug A are reduced significantly more, on average, than the blood pressures of the volunteers who are given Drug B. Explain why we have evidence that Drug A *causes* a greater reduction in blood pressure than Drug B.

2. **Explanatory and Response Variables; Levels, Treatments**

Suppose, now, that an experiment is designed to determine the effects of a particular drug and of exercising on people's blood pressures. Each participant will be given no drug at all, or 2mg per day of the drug, or 10 mg per day of the drug. Also, each participant will either exercise or not exercise. The reduction in blood pressure will be measured for each person who takes part in the experiment.

(a) What is/are the explanatory variable(s) in this experiment?

(b) What is/are the response variable(s) in this experiment?

(c) For each explanatory variable, how many levels are there?

(d) How many treatments are there?

(e) What/who are the experimental units in this context?

3. **Control Group**

Consider an experiment to determine whether regular exercise, over an extended period, reduces blood pressure. A substantial number of volunteers who suffer from high blood pressure and who do not, as yet, exercise regularly will be used in the experiment.

(a) Suppose that all the participants in the experiment are given daily sessions of supervised exercise over a period of four months, and that their blood pressures are, on average, significantly reduced. Explain why this result does not give us evidence that regular exercise reduces blood pressure.

(b) Suppose now that the experiment is adapted so that the participants are randomly assigned to two groups. The participants in one of the groups are given the supervised daily exercise, while those in the other group (the control group) do not exercise. How does the inclusion of the control group solve the problem described in part (a)?

4. **Placebo Effect, Placebo**

Consider an experiment in which a single drug, designed to reduce blood pressure, is being tested. The participants (all of whom suffer from high blood pressure) are randomly assigned to two groups: Group A and Group B. The participants in Group A will receive the drug.

(a) Suppose that the participants assigned to Group B are given no treatment at all, and that the participants in Group A undergo a significantly greater reduction in blood pressure than the participants in Group B. Explain why this does not give us evidence that the drug is effective.

(b) What, in the context of this experiment, is a placebo? Explain how use of a placebo for Group B overcomes the problem described in part (a) of this question.

5. **Lurking (Extraneous) Variables, Confounding Variables**

An experiment is designed to compare three different fertilizers, Fertilizer A, Fertilizer B, and Fertilizer C, for the growth of potted plants of a particular species. A number of very similar young plants of this species are planted in soil in identical pots. Some of the pots are treated using Fertilizer A, some using Fertilizer B, and some using Fertilizer C. At the end of the experiment the grown plants are compared using a measure of quality that includes considerations such as the number of flowers, the number of leaves, and other aspects of the health of the plant.

(a) What is the explanatory variable in this experiment?

(b) What is the response variable?

(c) A lurking (extraneous) variable is a variable that is not the explanatory variable (or the response variable), but that nonetheless might have an effect on the response variable. List three possible lurking variables in the context of this experiment.

(d) Choose one of the lurking variables you provided in part (c), and explain what would need to be the case for this variable to be described as a *confounding* variable.

6. **Generalization**

Return to the experiment described in Question 1, and assume that the 40 volunteers (who all suffered from high blood pressure) were randomly assigned to receive either Drug A or Drug B. Suppose that the blood pressures of the volunteers who were given Drug A were reduced significantly more, on average, than the blood pressures of those who were given Drug B. Can we conclude that Drug A would be more effective than Drug B for *all* patients who suffer from high blood pressure? Explain your answer.

7. **Describing an Experiment, Matched Pairs**

(a) It has been suggested that the application of lemon juice or vinegar to a sliced avocado can prevent discoloration. Suppose that you have been provided with 30 half avocadoes, recently cut. Describe a completely randomized experiment to determine which, of lemon juice or vinegar, is more effective for reducing discoloration. (Do not include a control group.)

(b) How might the experiment described in part (a) be adapted in order to make use of a matched pairs design? (Be sure to explain how the treatments would be assigned.)

(c) Why is a matched pairs design preferable to the design in part (a)?

8. **Blinding**

(a) Return, again, to the experiment described in Question 1 where a new drug (Drug A) is compared to a current drug (Drug B), and where both drugs are designed to reduce blood pressure. A number of volunteers who suffer from high blood pressure will be randomly assigned to receive either Drug A or Drug B.

(i) What two criteria are required for the experiment to be described as "double blind"?

(ii) Explain why the two criteria you provided in part (a) are important in this experiment.

(iii) What would need to be the case for the experiment to be described as "single blind"?

(b) Suppose an experiment were to be designed to determine the effect of regular exercise on people's blood pressure. Explain why a double blind design is not possible for this experiment.

9. **Replication**

A statistics teacher wants to compare three different teaching methods: Method A, Method B, and Method C. She decides to use her class of 18 students. Completely randomly, she will assign 6 students to Method A, 6 to Method B, and 6 to Method C. The students will be taught the same topic using these methods, and they will then all be given the same test. The three methods will be compared by comparing the average test results for the three groups.

What problem might arise as a result of using as few as 6 students in each treatment group? Explain why using a larger class of 42 students, for example (and therefore having 14 students in each treatment group), would be preferable.

10. **Blocking**

Return to the scenario described in the previous question, where the teacher wishes to compare the three teaching methods using her class of 18 students.

(a) The teacher has a list of all the students' average scores in her course up to the time when she is going to start the experiment. Explain how she would conduct this experiment using blocking by average grade in the course. Use blocks of size three, and be sure to include a detailed explanation as to how the treatments would be assigned.

(b) Explain why the block design is preferable to the completely randomized design described in the previous question.

PROBABILITY

1. A music enthusiast has a collection of recorded music consisting of 3390 albums. Each album has been classified as one of classical, jazz, or popular, and is on either CD, vinyl, or cassette. (There is only one recording of each album in the collection.) The numbers of albums falling into these categories are given in the table below.

	CD	Vinyl	Cassette	Total
Classical	846	690	81	1617
Jazz	693	562	116	1371
Popular	158	95	149	402
Total	1697	1347	346	3390

(a) An album is chosen at random from this collection. Find the probability that it is

(i) jazz

(ii) jazz and on vinyl

(iii) jazz or on vinyl

(iv) jazz, given that it is on vinyl

When an album is chosen at random from the collection, let J be the event that it is jazz and let V be the event that it is on vinyl.

(b) Are J and V mutually exclusive events?

(c) Are J and V independent events?

2. In a particular community, 80% of the people wear deodorant, 40% exercise regularly, and 84% do at least one of these two things.

(a) If a person is chosen at random from this community, what is the probability that the person wears deodorant and exercises regularly?

(b) If a person is chosen at random from this community, what is the probability that the person neither wears deodorant nor exercises regularly?

(c) What is the probability that a person in this community wears deodorant given that he/she exercises regularly?

(d) If a person is known not to wear deodorant, what is the probability that the person exercises regularly?

(e) In this community, are the events "wears deodorant" and "exercises regularly" mutually exclusive? Explain.

(f) In this community, are the events "wears deodorant" and "exercises regularly" independent? Explain.

(g) In a random sample of 1000 people from this community, how many would you expect to wear deodorant? Would exactly this number of people in the sample wear deodorant?

3. A student is about to take APs in US History, English Language, and Statistics. She estimates that her probabilities of getting 5's in these subjects are 0.6, 0.7, and 0.8, respectively. She is also willing to assume that her results in the three subjects are independent. Assuming that the student's estimates are correct, find the probability that she gets

   (a) 5's in all three subjects

   (b) no 5's

   (c) exactly one 5

   (d) at least one 5

4. A student named Lenny rides a bicycle to school on 3/5 of days, and on the other days is driven to school by car. When he uses the bicycle he is able to avoid traffic, and is on time to school with probability 0.95. When he is driven to school he is on time with probability 0.75.

   (a) Complete the tree diagram below by writing the specified probabilities in the boxes.

      (i) The probability that, on a randomly chosen day, Lenny rides his bicycle to school

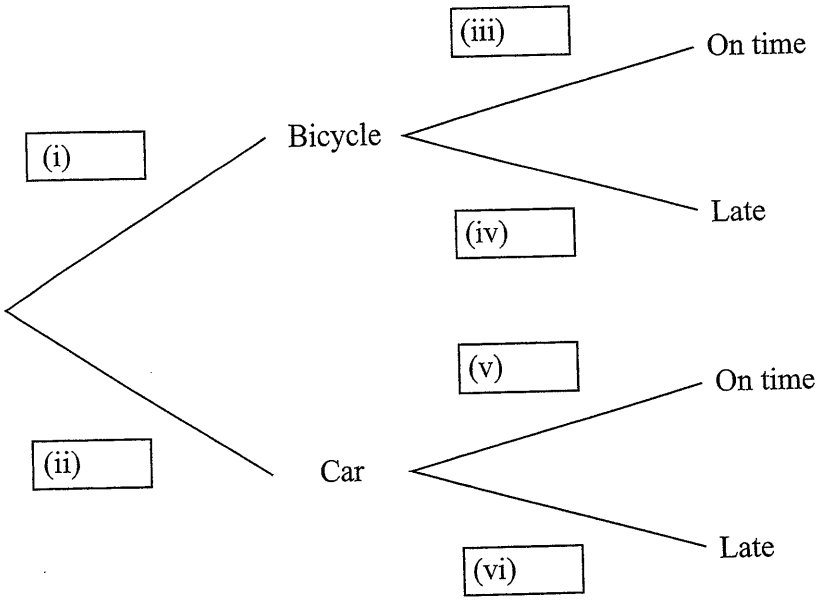
      (ii) The probability that, on a randomly chosen day, Lenny is driven to school

      (iii) The probability that Lenny is on time, given that he rides his bicycle to school

      (iv) The probability that Lenny is late, given that he rides his bicycle to school

      (v) The probability that Lenny is on time, given that he is driven to school

      (vi) The probability that Lenny is late, given that he is driven to school



(b) If a day is chosen at random, what is the probability that, on that day, Lenny rides his bicycle to school and is on time?

(c) On what proportion of days is Lenny on time?

(d) If, on a particular day, Lenny is observed to be on time, what is the probability that Lenny rode his bicycle to school on that day?

5. A statistics teacher has 7 girls and 5 boys in her class. If the teacher chooses three students at random to record the results of a study, what is the probability that all three students are girls?

RANDOM VARIABLES

1. As part of its census, a country keeps record of the number of vehicles available to each household. Using this information, it is calculated that, when a household is chosen at random from the country, the probability distribution of the number of vehicles available is as shown below.

Number of Vehicles	0	1	2	3
Probability	0.32	0.42	0.17	0.09

(a) Calculate the mean number of vehicles per household in this country.

(b) Calculate the standard deviation of the number of vehicles available to households.

(c) The unit of currency for the country is the guilder. The government imposes a monthly tax of 320 guilders per household, plus 180 guilders for each vehicle available to the household. Using your answers to parts (a) and (b), calculate the mean and the standard deviation of the monthly tax amount (in guilders) for a randomly chosen household.

2. Jenna has started a business selling half-liter bottles of spring water. She sells the water using two methods. The first is through a small stall set up at a popular tourist location, where she sells individual bottles for cash. The second is through a web site, where customers can order cases of the bottles for local delivery. The number of bottles that she sells from the stall on a randomly selected day has mean 128 and standard deviation 16. The number of bottles she sells via the web site on a randomly selected day has mean 223 and standard deviation 35.

   (a) Calculate the expected value of the total number of bottles sold on a randomly selected day.

   (b) What assumption do you need to make in order to use the information given above to calculate the standard deviation of the total number of bottles sold on a randomly selected day? Do you consider this assumption to be reasonable? Explain.

   In the questions that follow, assume that the assumption in part (b) holds.

   (c) Calculate the standard deviation of the total number of bottles sold on a randomly selected day.

   (d) Calculate the mean and the standard deviation of the amount by which, on a randomly selected day, the number of bottles sold through the web site exceeds the number of bottles sold from the stall.

   (e) Jenna charges \$1.25 per bottle at the stall and \$0.57 per bottle on the web site. Calculate the mean and the standard deviation of the total amount of money she takes on a randomly selected day.

3. Nick plays basketball. When Nick takes a free throw, the probability that he is successful is 0.7. Today, Nick will take six free throws. Assuming that the outcomes of the throws are independent of each other, find the probability that he has

   (a) exactly four successes

   (b) at least four successes

4. Nick's younger brother, James, also plays basketball. For James, the probability of being successful on a free throw is 0.35. Suppose that James will take 10 free throws and that the outcomes of the throws are independent of each other.

   (a) Find the probability that James has

      (i) no successes

      (ii) at least one success

      (iii) at least three successes

   (b) What are the mean and the standard deviation of the number of successes for James?

5. The basketball player in the previous question, James, decides to start taking free throws, and to continue until he gets his first success. Find the probability that the number of throws he takes up to and including his first success is

   (a) three

   (b) less than three

   (c) more than three

6. In a particular population of polar bears, the adult males have masses that are normally distributed with mean 515 kilograms and standard deviation 88 kilograms.

   (a) An adult male is chosen at random from this population. Calculate the probability that his mass is

      (i) between 480 and 580 kilograms

      (ii) less than 600 kilograms

      (iii) more than 450 kilograms

   (b) What is the minimum mass required for a bear to be amongst the heaviest 20% of adult males in this population?

7. For a population of polar bears, 22% of the adult females have masses less than 240 kilograms. If the masses of adult females are known to be normally distributed with standard deviation 51 kilograms, what is the mean mass of adult females in this population?

8. For each of the following say whether the distribution of the random variable X is most likely to be binomial, geometric, normal, or none of these. ("None of these" is allowed in only one answer.)

   (a) You stand on a street in New York City and watch people using their cell phones. X is the number of phones out of the next ten that are smart phones.

   (b) X is the number of flips of a coin until you get a head.

   (c) X is the length of the next French fry that you eat.

   (d) X is the number of journeys that you make to school up to the first viewing of a car with its trunk not fully closed.

   (e) X is the score when a cube with faces numbered 1 through 6 is rolled.

   (f) You know how many coins there are in your pocket. X is the number of heads that show when you drop all the coins on the floor.

MISCELLANEOUS HYPOTHESIS TESTING

As a first exercise, for each of the questions below, simply state the type of hypothesis test that is required.

As a second exercise, perform all the tests.

1. A large pine forest is populated by three different types of pine tree: Sand Pine, Shortleaf Pine, and Loblolly Pine. It is known that 58% of the trees in the forest are Sand Pines, 22% are Shortleaf Pines, and 20% are Loblolly Pines. A random sample of 100 trees is selected from one region of the forest, and these 100 trees are categorized according to type, with the results as shown below.

Type of Pine	Sand	Shortleaf	Loblolly
Number of Trees	55	14	11

   Does this sample provide convincing evidence that the proportions of the three types of pine are different in this region of the forest from the proportions in the forest as a whole?

2. The owners of a supermarket chain are interested in promoting fruit for its health-giving properties. As a pilot study, a random sample of 200 customers is selected, and each customer in the sample is asked whether he/she believes that eating oranges is good for your health. Of these 200 customers, 186 reply "Yes," with the remainder replying "No." Does this result provide convincing evidence that more than 90% of all customers would answer "Yes" to the question?

3. Ten years ago, a survey was conducted at a large college using a random sample of 140 students. Each student was asked "If you get a poor grade and have to tell a parent, do you tell him/her by email, by phone, or do you leave it until you next see him/her in person?" A similar survey was conducted at the same college five years ago using a random sample of 119 students, and again, this year, using a random sample of 132 students. The responses to the three surveys are summarized in the table below.

	10 Years Ago	5 Years Ago	This Year
Email	13	44	64
Phone	90	45	43
In Person	37	30	25

Do these results provide convincing evidence of any difference between the populations for the three different years in terms of the distribution of preferred mode of communication?

4. At a large high school for academically talented students, a random sample of 15 students was selected to take a mathematical problem solving contest in two consecutive years. The scores for the 15 students are given below.

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Last Year	84	80	95	83	77	84	55	86	95	102	72	96	80	87	64
This Year	91	89	79	81	91	81	74	101	107	90	60	98	85	65	99

Do these results provide convincing evidence that, if all the students at the school had taken the contest on both occasions, this year's mean score would have been greater than last year's?

5. Do the results in the previous question provide convincing evidence of a useful linear relationship between last year's scores and this year's scores?

6. At a different school from the one described in question 4, a random sample of 10 students was selected to take the math contest last year, and a different random sample of 10 students was selected to take the contest this year. The results are shown below.

Last Year	74	60	82	82	104	93	73	94	82	94
This Year	94	93	111	88	99	97	96	101	73	84

Do these results provide convincing evidence that, if all the students at the school had taken the contest on both occasions, this year's mean score would have been greater than last year's?

7. Refer to the results given in the previous question. Do last year's results provide convincing evidence that, if all the students at this school had taken the contest last year, the mean score would have been less than 90?

8. In a random sample of 165 men in long-term relationships, 32 said that they had bought valentines cards for their partners. In a random sample of 178 women in long-term relationships, 52 said that they had bought valentines cards for their partners. Do these results provide sufficient evidence to conclude that men and women in long-term relationships are different in terms of the proportions who would say that they bought valentines cards for their partners?

9. A large school district offers two buses on all its routes: an early bus and a late bus. On any given morning the students are free to decide which bus to take. Some take the early bus, as it covers the route more quickly and enables the student to take part in morning activities; others take the late bus as it gives them a small amount of extra sleep. On a particular day, a random sample of 350 high school students was selected. Each student was asked which bus he/she took, and the student's grade level was noted. The numbers of students falling into the various categories were as shown below.

	9	10	11	12
Early	56	48	38	83
Late	39	52	46	64

Do these results provide convincing evidence of an association between grade level and choice of bus for high school students in the district?

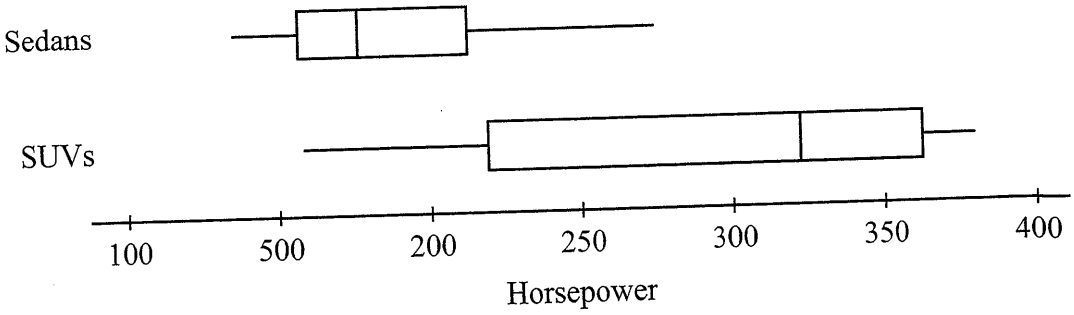
Sample Examination One

SECTION I

Time—1 hour and 30 minutes
Number of questions—40
Percent of total grade—50

Directions: Solve each of the following problems, using the available space for scratch work. Decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

1. A used car salesman deals in sedans and SUVs, and keeps note of the horsepower of each vehicle. The values for his current inventory are represented by the boxplots below.



Which of the following is true?

(A) The distribution for the sedans is positively skewed and the distribution for the SUVs is negatively skewed. The range is greater for the sedans than for the SUVs.
(B) The distribution for the sedans is negatively skewed and the distribution for the SUVs is positively skewed. The range is greater for the sedans than for the SUVs.
(C) The distribution for the sedans is positively skewed and the distribution for the SUVs is negatively skewed. The range is less for the sedans than for the SUVs.
(D) The distribution for the sedans is negatively skewed and the distribution for the SUVs is positively skewed. The range is less for the sedans than for the SUVs.
(E) Both distributions are roughly symmetrical. The range is greater for the sedans than for the SUVs.

Answer

2. In order to study the effects of organic feed on the health of animals, some cows from a herd are randomly selected to be given organic feed, while the remaining cows are given a non-organic equivalent. At the end of the study the health levels of all the cows are measured. Which of the following is true?

(A) This is an observational study in which the level of health is the explanatory variable and the type of feed is the response variable.
(B) This is an observational study in which the type of feed is the explanatory variable and the level of health is the response variable.
(C) This is an observational study that could establish whether organic feed causes good health.
(D) This is an experiment in which the level of health is the explanatory variable and the type of feed is the response variable.
(E) This is an experiment in which the type of feed is the explanatory variable and the level of health is the response variable.

Answer

3. Of the male students at a high school, 35% play football, 44% play basketball, and 12% play both of these sports. If a male student is chosen at random, what is the probability that he plays exactly one of the sports?

(A) 0.482 (B) 0.55 (C) 0.636 (D) 0.67 (E) 0.79

Answer

4. A confidence interval will be used to estimate a population proportion. If a random sample of size 50 and a random sample of size 200 are selected, and 90% and 95% confidence intervals for the population proportion are calculated for each sample, which of the four confidence intervals is likely to be the narrowest?

(A) The 90% confidence interval for the smaller sample
(B) The 95% confidence interval for the smaller sample
(C) The 90% confidence interval for the larger sample
(D) The 95% confidence interval for the larger sample
(E) The two 95% confidence intervals, they being likely to have roughly equal widths

Answer

5. For a set of 15 decathletes, the correlation between their times for the 100 meter sprint and their distances in the long jump was -0.675. The standard deviation of their 100 meter times was 0.383 seconds and the standard deviation of their long jump distances was 0.469 meters. Denoting 100 meter time by x and long jump distance by y, what is the slope of the least squares regression line of y on x?

(A) -1.012 (B) -0.827 (C) -0.675 (D) -0.551 (E) -0.450

Answer

6. A polling organization is given the job of assessing whether the proportions of homemakers using various types of cooking oil are changing during an advertising campaign. A random sample of homemakers is selected before the campaign starts, and each homemaker in the sample is asked whether he/she primarily uses canola oil, olive oil, sunflower oil, or some other sort of oil for cooking. After the first stage of the campaign, a new random sample of homemakers is selected, and the people selected are asked the same question as those in the first sample. After the final stage of the campaign, a third random sample of homemakers is selected, and the people selected are asked the same question. Which of the following would be most suitable for analysis of the results of this study?

(A) One-sample z-test for a proportion
(B) Two-sample t-test for means
(C) Paired t-test
(D) Chi-square test for goodness of fit
(E) Chi-square test for homogeneity

Answer

7. In an election, the Democratic candidate received 799,072 votes, the Republican candidate received 783,426 votes, and the other two candidates received a combined total of 157,302 votes. These results could be appropriately represented using

(A) a stemplot
(B) a histogram
(C) a pie chart
(D) a boxplot
(E) a scatterplot

Answer

8. It is known that 68% of the adult residents of a large town are male, and that 90% of the adult male residents are employed and 76% of the adult female residents are employed. A random sample of 800 adult residents of the town is selected. Which of the following is closest to the expected number of people in the sample who are employed?

(A) 644 (B) 654 (C) 664 (D) 674 (E) 684

Answer

9. A particular ski slope is used by a large number of people. A researcher wishes to establish whether there is a difference between the mean times taken to complete the descent for men and women. In order to answer this question, independent random samples of men and women using the slope are selected and the times are recorded for the people selected. Let μ_M represent the mean time for all men and let μ_W represent the mean time for all women. If the results of the study are to be analyzed using a hypothesis test, what hypotheses should be used?

(A) $H_0: \mu_M = \mu_W, H_a: \mu_M \neq \mu_W$
(B) $H_0: \mu_M = \mu_W, H_a: \mu_M > \mu_W$
(C) $H_0: \mu_D = 0, H_a: \mu_D \neq 0$
(D) $H_0: \mu_D = 0, H_a: \mu_D > 0$
(E) $H_0: \mu_M > \mu_W, H_a: \mu_M < \mu_W$

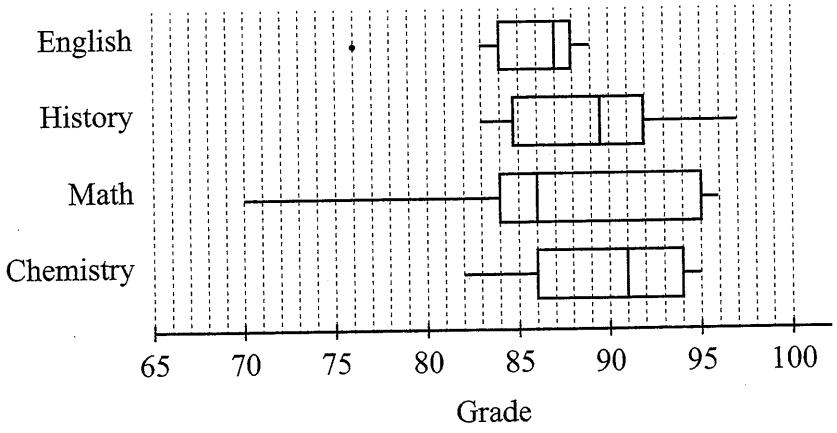
Answer

10. Garden Delicious pea pods have lengths that are approximately normally distributed with standard deviation 0.8 inches. The largest 1 percent of pods are eligible for prizes. How many inches above the mean pod length is the smallest pod that is eligible for a prize?

(A) 1.64 (B) 1.86 (C) 1.94 (D) 2.02 (E) 2.33

Answer

11. Larry is enrolled in four academic classes. The grades for all the students in Larry's four classes are represented by the boxplots shown below. (The boxplot for the students in Larry's English class shows one outlier, represented by a dot.)



Larry's grades are 84 in English, 88 in History, 86 in Math, and 92 in Chemistry. His advisor decides to list the four classes according to how well Larry performed relative to his fellow students, with the class in which Larry performed best (relative to his fellow students) showing first in the list. Which of the following is the correct list?

(A) Chemistry, English, Math, History
(B) Math, Chemistry, English, History
(C) Chemistry, Math, English, History
(D) Math, Chemistry, History, English
(E) Chemistry, Math, History, English

Answer

12. A hypothesis test is performed using the results of a random sample from a large population. The test is based on the null hypothesis, H_0, that a population parameter takes a particular value, and the alternative hypothesis, H_a, that the parameter does not take that value. Which of the following is true?

(A) The results could provide convincing evidence that the null hypothesis is true.
(B) The results could provide convincing evidence that the alternative hypothesis is true.
(C) The results could prove that the alternative hypothesis is false.
(D) The null hypothesis can only be rejected if it is false.
(E) Failure to reject the null hypothesis means that the null hypothesis is true.

Answer

13. Suppose that it is known that 13% of people are left-handed. If ten people are chosen at random, what is the probability that exactly two of them are left-handed?

(A) $\frac{1}{10}\binom{10}{2}$
(B) $(0.13)^2(0.87)^8$
(C) $(0.87)^2(0.13)^8$
(D) $\binom{10}{2}(0.13)^2(0.87)^8$
(E) $\binom{10}{2}(0.87)^2(0.13)^8$

Answer

14. A large Internet-based company serving the USA wishes to send a survey to a sample of its customers. Which of the following will result in a stratified random sample?

(A) Sending the survey to the next 4000 customers who place orders
(B) Numbering a complete list of customers sequentially and using a computer to randomly select 4000 customers from the list
(C) Numbering a complete list of customers sequentially and sending the survey to the customers numbered 258, 1258, 2258, 3258, and so on
(D) Dividing the country into a large number of regions, randomly selecting 30 of those regions, and sending the survey to all the customers in those 30 regions
(E) Dividing the customers into four separate groups according to the type of goods primarily ordered, and randomly selecting 1000 customers from each group

Answer

15. For a classroom activity, a teacher uses a bag containing 300 blue chips and 200 red chips. The teacher demonstrates the process of picking ten chips at random from the bag (replacing the chips and mixing between picks) and calculating the proportion of the ten chips that are blue. The students then repeat this process a large number of times, keeping note of the proportion of the ten chips that are blue on each occasion. The standard deviation of all the proportions calculated is likely to be closest to which of the following?

(A) $\sqrt{\frac{(0.6)(0.4)}{10}}$
(B) $\sqrt{10(0.6)(0.4)}$
(C) $\sqrt{\frac{(0.6)(0.4)}{500}}$
(D) $\sqrt{500(0.6)(0.4)}$
(E) $\frac{(0.6)(0.4)}{\sqrt{500}}$

Answer

16. A random sample of size 8 has been selected from a large population, and the sample mean, $\overline{x}$, and the sample standard deviation, s, have been calculated. The population standard deviation is unknown. A confidence interval for the population mean is to be constructed. What is the correct formula to use, and what assumption has to be made about the population?

(A) $\overline{x} \pm z^* \cdot \frac{\sigma}{\sqrt{8}}$; no assumption about the about the population is necessary

(B) $\overline{x} \pm z^* \cdot \frac{s}{\sqrt{8}}$; we have to assume that the population is normally distributed

(C) $\overline{x} \pm t^* \cdot \frac{\sigma}{\sqrt{8}}$; no assumption about the about the population is necessary

(D) $\overline{x} \pm t^* \cdot \frac{s}{\sqrt{8}}$; no assumption about the about the population is necessary

(E) $\overline{x} \pm t^* \cdot \frac{s}{\sqrt{8}}$; we have to assume that the population is normally distributed

Answer

17. Frequently, experiments are designed to take account of the fact that many people show improvement resulting purely from the psychological effect of taking tablets, even if the tablets contain no active ingredient.

In an experiment, the subjects are randomly assigned to two groups. The people in one of the groups (the "treatment group") are given tablets containing a new drug. The people in the other group (the "placebo group") are given tablets that look and taste exactly the same as the other tablets, but contain no active ingredient. In order that the experiment should test the effectiveness of the drug, the experimental design depends on the fact that

(A) subjects in both groups could experience the potentially positive psychological effect of taking tablets
(B) only subjects in the placebo group could experience the potentially positive psychological effect of taking tablets
(C) only subjects in the treatment group could experience the potentially positive psychological effect of taking tablets
(D) no subject in the experiment will experience the potentially positive psychological effect of taking tablets
(E) nobody could ever experience a positive psychological effect as a result of taking tablets

Answer

Questions 18 and 19 refer to the following scenario and numerical information.

Eighty runners took part in a cross country race. Their times are summarized in the table below.

Time (minutes)	16	18	20	22	24	26	28	30	32
Cumulative Relative Frequency	0.000	0.0750	0.2625	0.5125	0.7125	0.8250	0.9375	0.9750	1.000

(The cumulative relative frequencies refer to the proportions of runners whose times were less than or equal to the times given.)

18. How many runners had times that were more than 20 minutes and at most 22 minutes?

(A) 10 (B) 20 (C) 30 (D) 40 (E) 50

Answer

19. Which of the following could be the interquartile range of the times?

(A) 3 minutes, 50 seconds
(B) 4 minutes, 40 seconds
(C) 8 minutes, 20 seconds
(D) 10 minutes, 30 seconds
(E) 12 minutes, 10 seconds

Answer

20. A random sample of 50 Brand A light bulbs and an independent random sample of 45 Brand B light bulbs were selected, and the lives (in hours) of the bulbs in the samples were measured. The partial computer output below shows the results of a test of H_0: $\mu_A = \mu_B$ *versus* H_a: $\mu_A \neq \mu_B$.

```
Two-sample T for Brand A vs Brand B

          N     Mean       StDev      SE Mean
Brand A   50    907.6      60.2       8.5
Brand B   45    890.8      46.0       6.9

Difference = mu (Brand A) - mu (Brand B)

T-Test of difference = 0 (vs not =):
T-Value = 1.54    P-Value = 0.127    DF = 90
```

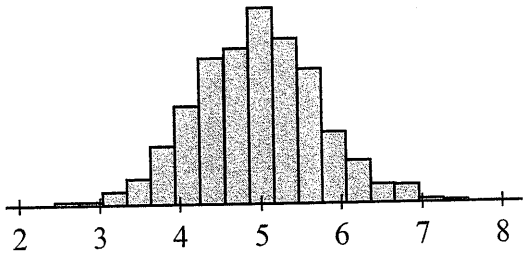
Which of the following is NOT true?

(A) H_0 is not rejected at the 0.05 significance level.
(B) The value 1.54 is less than the positive critical value of a t distribution with 90 degrees of freedom for a single-tail probability of 0.025.
(C) A 95% two-sample t confidence interval based on these results would contain zero.
(D) If the population means were equal, the probability of getting a t statistic whose absolute value is at least 1.54 would be 0.127.
(E) Given a difference in sample means of 16.8, the probability that the population means are equal is 0.127.

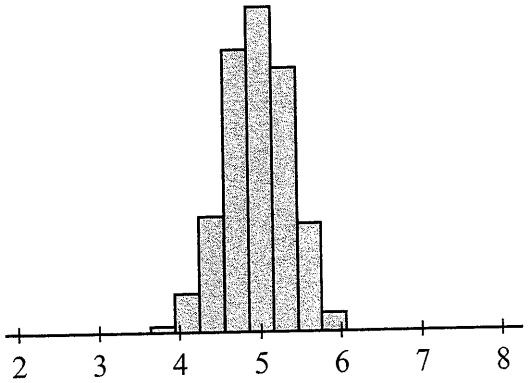
Answer

21. Three statistics, Statistic 1, Statistic 2, and Statistic 3, are to be compared as estimators of a particular population parameter. To estimate the behavior of the statistics, 600 random samples are selected from the population, and the value of each statistic is calculated for each sample. The true value of the population parameter is 5. The distributions of the values of the three statistics are shown in the graphs below.

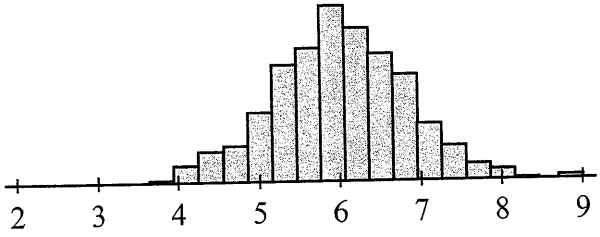
Statistic 1



Statistic 2



Statistic 3



The three statistics are to be listed in order of preference, with the best statistic first in the list. Which of the following is correct?

(A) 1, 2, 3 (B) 2, 1, 3 (C) 1, 3, 2 (D) 3, 1, 2 (E) 2, 3, 1

Answer

22. A simple random sample of size 50 is selected from a population, and a measurement is taken for each individual in the sample. These results will be used to test the null hypothesis H_0: $\mu = 8$ *versus* the alternative hypothesis H_a: $\mu > 8$. A significance level of $\alpha = 0.05$ will be used for the test. Assuming that the true value of the population mean, μ, is greater than 8, which of the following would produce a test that has greater power than the one given above?

I. Changing the significance level to $\alpha = 0.1$
II. Changing the alternative hypothesis to H_a: $\mu \neq 8$
III. Increasing the sample size to 100

(A) I only (B) II only (C) III only (D) I and III (E) II and III

Answer

23. A survey is to be designed in order to estimate some quantities associated with a population. Which of the following is NOT true?

(A) A census will always be more representative of the population than a sample.
(B) How well a sample will represent the population is influenced by the quality of the sampling method used.
(C) How well a random sample will represent the population is partly a matter of chance.
(D) A simple random sample will always represent the population better than a systematic sample.
(E) A convenience sample is unlikely to be representative of the population.

Answer

24. In a particular state, it is known that 40% of daily trips are for shopping or running errands, 30% are for social or recreational purposes, 18% are for commuting to work, and 12% are for other purposes. A survey in one town within the state included 500 daily trips, of which 192 were for shopping or running errands, 133 were for social or recreational purposes, 118 were for commuting to work, and 57 were for other purposes. A hypothesis test is conducted in order to find out whether the pattern of daily trips for the town differs from the pattern for the state as a whole. (The test is based on the assumption that the 500 daily trips in the survey form a random sample of the daily trips for the town.) What is the value of the test statistic?

(A) 1.59 (B) 5.03 (C) 8.23 (D) 9.31 (E) 11.11

Answer

25. It is known that one-fifth of the vehicles that pass a particular intersection are commercial vehicles, and that the vehicles pass this intersection independently. A student is planning to stand at the intersection and count the vehicles that pass up to and including the first commercial vehicle. Which of the following best describes the distribution of the number of vehicles the student will count?

(A) Binomial
(B) Chi-square
(C) Geometric
(D) Normal
(E) *t*

Answer

26. The management of a factory wishes to compare a new machine for producing saucers with the machine the factory currently uses for that purpose. Random samples of 80 saucers from the new machine and 100 saucers from the current machine are selected, and it is found that 17 of the saucers from the new machine and 28 of the saucers from the current machine have faults. The new machine will be incorporated if, and only if, these results provide convincing evidence at the 0.05 significance level that the proportion of faulty saucers is less for the new machine than for the current machine. Will the new machine be incorporated?

(A) Yes, because $P\left(z < \dfrac{0.2125 - 0.28}{\sqrt{\dfrac{(0.2125)(0.7875)}{80} + \dfrac{(0.28)(0.72)}{100}}}\right)$ is greater than 0.05.

(B) No, because $P\left(z < \dfrac{0.2125 - 0.28}{\sqrt{\dfrac{(0.2125)(0.7875)}{80} + \dfrac{(0.28)(0.72)}{100}}}\right)$ is greater than 0.05.

(C) Yes, because $P\left(z < \dfrac{0.2125 - 0.28}{\sqrt{(0.25)(0.75)\left(\dfrac{1}{80} + \dfrac{1}{100}\right)}}\right)$ is greater than 0.05.

(D) No, because $P\left(z < \dfrac{0.2125 - 0.28}{\sqrt{(0.25)(0.75)\left(\dfrac{1}{80} + \dfrac{1}{100}\right)}}\right)$ is greater than 0.05.

(E) Yes, because $P\left(z < \dfrac{0.2125 - 0.28}{\sqrt{(0.25)(0.75)\left(\dfrac{1}{80} + \dfrac{1}{100}\right)}}\right)$ is less than 0.05.

Answer

Questions 27 and 28 refer to the following scenario and numerical information.

The department of transportation for a particular state kept records of the number of new cars sold (x) and the number of used cars sold (y) for each month last year. Some computer output from a regression analysis of these data is shown below.

Dependent variable: Number of used cars sold

Predictor	Coef	StDev	T	P
Constant	32971	9656	3.41	0.007
New_cars	0.8566	0.4593	1.86	0.092

S = 5418.57 R-Sq = 25.8% R-Sq(adj) = 18.4%

27. During the month of July, 22,836 new cars and 57,693 used cars were sold. What is the residual for this data point?

(A) 5161 (B) −5161 (C) 5323 (D) −5323 (E) −5774

Answer

28. Treating the twelve months last year as a random sample of all months, and assuming that the other conditions for inference are met, at what level of significance do last year's results provide evidence of a non-zero slope in the population regression line of y on x?

(A) At the 0.01 level
(B) At the 0.05 level, but not at the 0.01 level
(C) At the 0.1 level, but not at the 0.05 level
(D) At the 0.05 level, but not at the 0.1 level
(E) Not at any reasonable significance level

Answer

29. Which of the following could be conducted in a double-blind manner?

(A) An experiment to investigate whether listening to music while typing increases the number of errors made
(B) An experiment to investigate whether regular exercise reduces blood pressure
(C) An experiment to investigate whether taking vitamin C speeds recovery from a cold
(D) An experiment to investigate whether drinking sufficient quantities of water increases the effectiveness of food supplements
(E) An experiment to investigate whether use of keyboard shortcuts reduces the time taken to perform a particular computer task

Answer

30. A transportation authority conducts a survey of users of a commuter railroad. A random sample of passengers is selected, and each passenger in the sample is given a questionnaire. There are two questions on the questionnaire. The first question asks whether the passenger paid for the journey at the ticket office, using a machine located on the platform, or online. The second question asks how happy the passenger is with the transportation service (very happy, happy, neutral, or unhappy). A hypothesis test will be conducted to determine whether there is an association between the method of payment and happiness with the service. The test will use a chi-square distribution with k degrees of freedom. What is the value of k?

(A) 2 (B) 3 (C) 6 (D) 9 (E) 12

Answer

31. After a successful year, a company decides to increase the salaries of all of its employees by 5 percent. Which of the following will NOT be increased by 5 percent?

(A) The mean salary
(B) The standard deviation of the salaries
(C) The variance of the salaries
(D) The median salary
(E) The interquartile range of the salaries

Answer

32. An experiment is to be designed to compare the side-effects associated with a new drug with those associated with a current drug designed for the same purpose. It is accepted that the older a person is, the more likely it is that the person will be negatively affected by these drugs. The designers of the experiment therefore decide to block by age. This blocking will ensure that

(A) all the older people will receive one of the drugs, with the younger people receiving the other drug
(B) in terms of age, the people who receive the new drug are different from the people who receive the current drug
(C) in terms of age, the people who receive the new drug are similar to the people who receive the current drug
(D) the people who take the current drug will be similar to each other with respect to age, and the people who take the new drug will be similar to each other with respect to age
(E) the assignment of the subjects to the two drugs is completely random

Answer

33. In the context of linear regression, an influential point is a data point whose removal would have a large effect on the least squares regression line of y on x. For reasonably large data sets, which of the following are true?

I. Any point with a large residual is an influential point.
II. Any point that is an outlier in the x-direction is an influential point.
III. Removal of an influential point could increase the absolute value of the correlation coefficient.

(A) II only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

Answer

34. It is estimated that, for the people in a large community, the standard deviation of the daily calorie intake is 245. Assuming that this standard deviation is correct, how large a random sample of people from the community would be necessary in order to estimate the mean daily calorie intake to within 30 calories with 95% confidence?

(A) 17 (B) 131 (C) 257 (D) 308 (E) 3922

Answer

35. Two blue cubes and three green cubes, each with faces labeled 1–6, will be rolled. Letting X be the total score for the two blue cubes and Y be the total score for the three green cubes, it can be shown that the standard deviations of the random variables X and Y are 2.42 and 2.96, respectively. Which of the following is the standard deviation of $X - Y$?

(A) −2.91 (B) −0.54 (C) 3.82 (D) 4.68 (E) 14.62

Answer

36. A statistics question requires a significance test with null hypothesis H_0: $p = 0.3$, where p is a population proportion. Two students, Juan and Tamara, both do the question, and they both calculate the correct positive value of the z-statistic. However, Juan performs a one-tailed test (using the alternative hypothesis H_a: $p > 0.3$), and Tamara performs a two-tailed test (using the alternative hypothesis H_a: $p \neq 0.3$). Given that both students are correct in their work, which of the following is NOT possible?

(A) Both students reject the null hypothesis at the 0.05 significance level.
(B) Both students fail to reject the null hypothesis at the 0.05 significance level.
(C) Juan rejects the null hypothesis at the 0.05 significance level and Tamara fails to reject the null hypothesis at the 0.05 significance level.
(D) Juan fails to reject the null hypothesis at the 0.05 significance level and Tamara rejects the null hypothesis at the 0.05 significance level.
(E) Juan fails to reject the null hypothesis at the 0.01 significance level and Tamara rejects the null hypothesis at the 0.05 significance level.

Answer

37. A random sample will be selected from a population of rabbits. The weights of the rabbits in the sample will be measured, and the sample mean weight will be calculated. Assuming that the sample size is greater than 1, the standard deviation of the sampling distribution of the sample mean is

(A) less than the population standard deviation, because the weight of any rabbit in the sample is likely to be closer to the population mean than the weight of a rabbit chosen at random from the population
(B) less than the population standard deviation, because, when calculating the sample mean, weights in the sample far from the population mean are averaged out with the other weights in the sample
(C) equal to the population standard deviation
(D) more than the population standard deviation, because there is a greater possibility of getting a weight that is far from the population mean in the sample than when picking one rabbit at random
(E) more than the population standard deviation, because using a large sample introduces a greater possibility of the sample mean being far from the population mean

Answer

38. A statistics teacher calculates the correlation between the heights (in inches) and the weights (in pounds) of the students in her class. Which of the following would change the value of the correlation coefficient?

(A) Adding 5 inches to each height
(B) Subtracting 10 pounds from each weight
(C) Adding 5 inches to each height and subtracting 10 pounds from each weight
(D) Subtracting each height from 100
(E) Converting the heights to centimeters

Answer

39. An article describes a t-test of $H_0: \mu = 20$ *versus* $H_a: \mu > 20$, and the value of the test statistic is given to be 1.58. The sample size is not given in the article, but is known to be between 5 and 30 inclusive. Assuming that all the conditions for inference are met, the p-value for the test would be

(A) less than 0.01
(B) between 0.01 and 0.05
(C) between 0.05 and 0.1
(D) between 0.1 and 0.2
(E) greater than 0.2

Answer

40. The ethnic breakdown of a particular county is known to be as follows: Caucasian: 38%, Hispanic: 34%, African American: 16%, and Asian: 12%. If two people are chosen at random from this county, what is the approximate probability that they are of the same ethnicity? (The population of the county can be assumed to be large.)

(A) 0.28 (B) 0.30 (C) 0.32 (D) 0.34 (E) 0.36

Answer

SECTION II
Part A

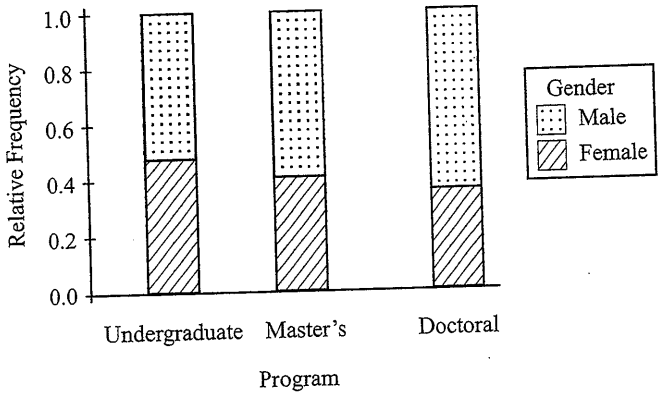
Questions 1–5

Spend about 65 minutes on this part of the exam.

Percent of Section II grade—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your method as well as on the accuracy and completeness of your results and explanations.

1. The graphical display below shows the proportions of male and female students in the undergraduate, master's, and doctoral programs at a particular university. These are the only three programs available at this university.



(a) What does the display reveal about the way the gender balance varies between the three programs?

(b) Does the information in the display imply that the number of males in the doctoral program is greater than the number of males in the masters program? Explain.

(c) When selecting a student at random from the university, are the events "is enrolled in the doctoral program" and "is female" mutually exclusive? Justify your answer.

(d) When selecting a student at random from the university, are the events "is enrolled in the doctoral program" and "is female" independent? Justify your answer.

2. A study concluded that AP students who attend extra study sessions do better, on average, on their AP exams than students who do not attend extra study sessions.

   (a) Explain why we cannot conclude from the result of the study that attending extra study sessions *causes* increased AP scores. Include an example of a plausible confounding variable.

An experiment is designed where 200 AP students will be randomly assigned to two groups (Group A and Group B), each of size 100. The students in Group A will be required to attend extra study sessions, while those in Group B will not be given that opportunity.

(b) Explain how the 200 students might be randomly assigned to the two treatment groups.

(c) Suppose that the students in Group A receive significantly higher AP results, on average, than the students in Group B. Explain why we now *do* have evidence that attending extra study sessions increases AP scores.

3. For every household in a particular county, the water use (in thousands of gallons) over the course of a year was recorded. The mean water use for the households in the county was found to be 162 and the standard deviation was 140.

   (a) Based on the information given above, could the distribution of household water use for that county be approximately normal? Explain your answer.

   (b) A random sample of 50 households will be selected, and the mean water use will be calculated for the households in the sample. Is the sampling distribution of the sample mean for random samples of size 50 approximately normal? Explain.

   (c) Suppose that the annual indoor water use (in thousands of gallons) for the same county is approximately normally distributed with mean 57 and standard deviation 12. If a random sample of 50 households is selected, what is the probability that their mean indoor water use (in thousands of gallons) will be greater than 59?

4. Random samples of 48 girls and 45 boys were selected from a large school district. It was found that 37 of the girls and 20 of the boys were not consuming the recommended amount of vitamin A.

   (a) Use a 95% confidence interval to estimate the difference between the proportions of girls and boys in the school district who do not consume the recommended amount of vitamin A.

   (b) Based only on this confidence interval, do you think that there is a difference between the proportions of girls and boys in the district who are not consuming the recommended amount of vitamin A? Justify your answer.

5. Exposure of workers to asbestos at construction sites and shipyards is considered dangerous. The workers at a construction site are concerned that asbestos might be present in the air, and so an inspector has been called. The inspector will select a random sample of locations at the site and will measure the asbestos level at those locations. If the data collected by the inspector provide convincing evidence that mean level of asbestos at the site is below the permissible exposure limit of 0.1 fibers per cubic centimeter (f/cc) then work at the site will be allowed to continue. Otherwise, work will stop until precautions have been put into place.

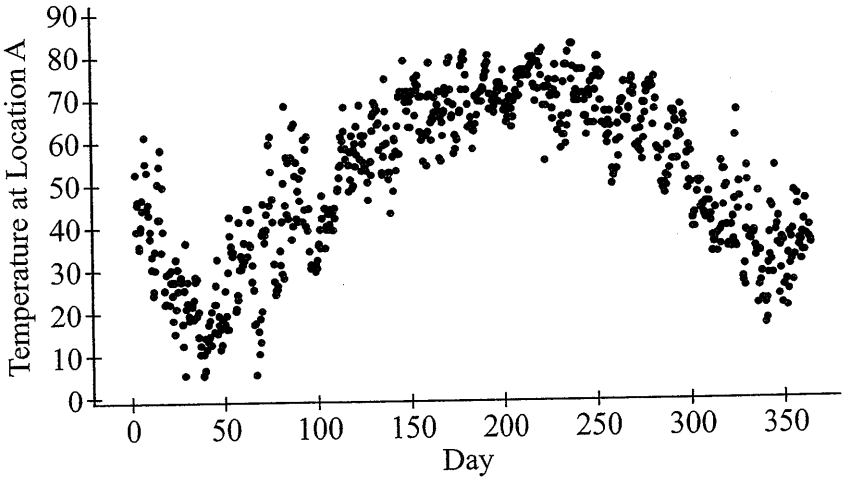
   (a) The results of the inspection will be analyzed by means of a hypothesis test. State the null and alternative hypotheses that would be used for the test, and define the parameter of interest.

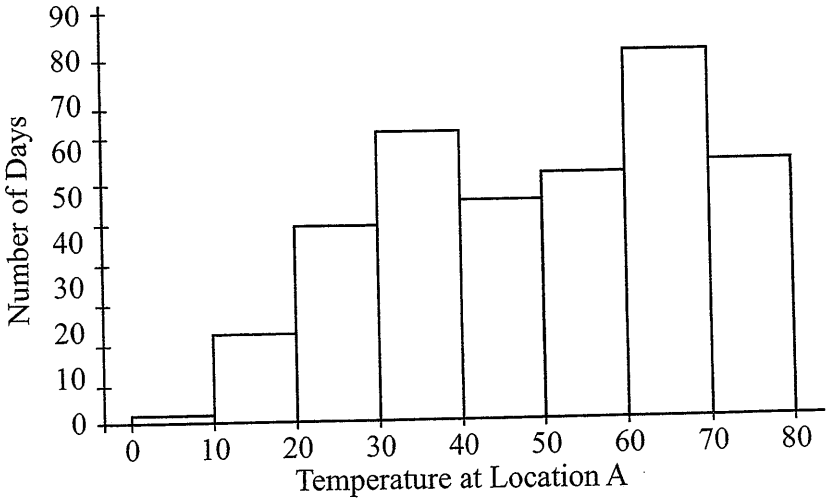
   (b) In the context of this situation, describe Type I and Type II errors and describe the consequences for the workers of each type of error.

SECTION II
Part B
Question 6
Spend about 25 minutes on this part of the exam.
Percent of Section II grade—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your method as well as on the accuracy and completeness of your results and explanations.

6. A meteorological organization records the daily temperatures at noon (local time) at various locations across the world. A time plot for the temperatures at Location A last year is shown below. (The days of the year are numbered 1 through 365, with "1" representing January 1st, and "365" representing December 31st. The days are shown on the horizontal axis, and, for each day, a single point is plotted showing the temperature for that day.) Additionally, a histogram summarizing the Location A temperatures for the same year is provided.

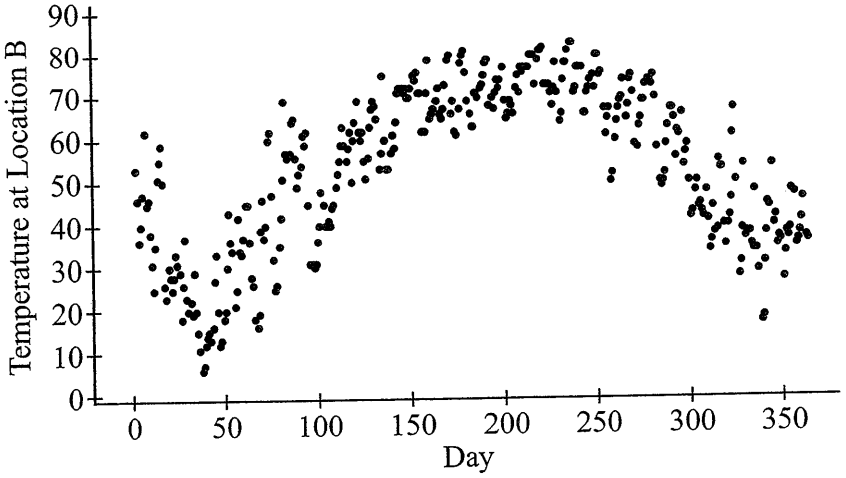




(a) Give two facts that are obvious from the histogram but are not obvious from the time plot.

(b) Give one fact that is obvious from the time plot but is not obvious from the histogram.

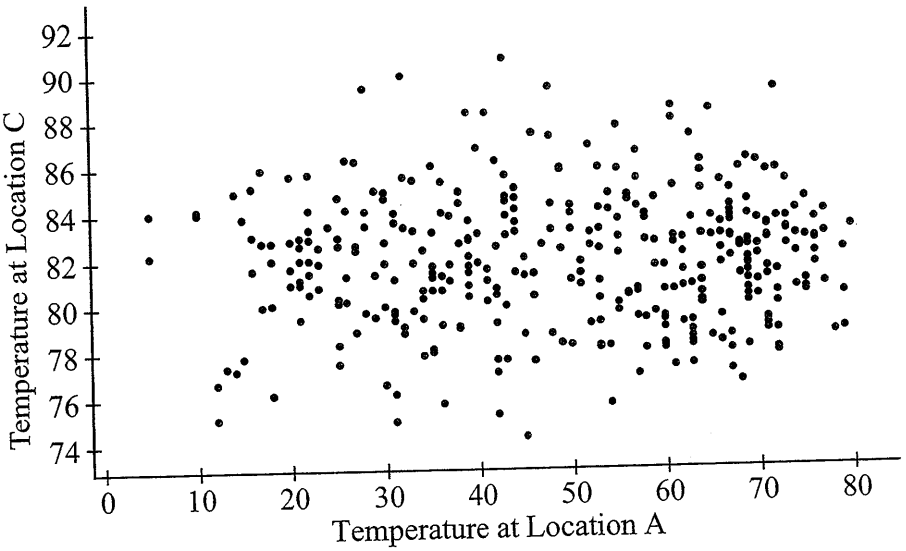
A time plot showing the temperatures at a different location, Location B, during the same year is shown below.



(c) A scatterplot will be constructed where, for each day in the given year, the temperature at Location A will be plotted as the x-coordinate and the temperature at Location B will be plotted as the y-coordinate. Make a rough sketch as to what the appearance of this scatterplot would be. Include approximate scales on your axes, but do not attempt to plot points exactly.

(d) Locations A and B are close geographically. A researcher wishes to establish whether, on average, the temperature at Location B is higher than at Location A. The researcher is willing to treat the days in the year considered above as a random sample of all days at the two locations. Name a hypothesis test that would be appropriate for answering the researcher's question.

The scatterplot below summarizes the temperatures at Location A and at a new location, Location C, for the days of the same year.



(e) Make a rough sketch of a possible appearance of the time plot for the temperatures at Location C during that same year. Include rough scales for your axes, but do not attempt to plot points exactly.

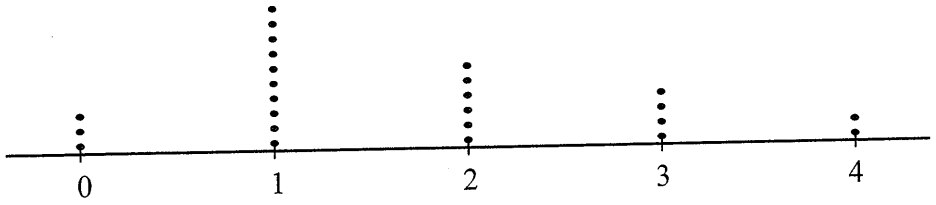
Sample Examination Two

SECTION I

Time—1 hour and 30 minutes
Number of questions—40
Percent of total grade—50

Directions: Solve each of the following problems, using the available space for scratch work. Decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

1. The dotplot below shows the number of goals scored by the Gator Girls soccer team in their games last year.



Number of Goals Scored

Which of the following could be used to compute the mean number of goals scored per game?

(A) $\frac{1+2}{2}$

(B) $\frac{0+1+2+3+4}{5}$

(C) $\frac{3+10+6+4+2}{5}$

(D) $\frac{3\cdot 0+10\cdot 1+6\cdot 2+4\cdot 3+2\cdot 4}{5}$

(E) $\frac{3\cdot 0+10\cdot 1+6\cdot 2+4\cdot 3+2\cdot 4}{3+10+6+4+2}$

Answer

2. There is concern that carrying weights while walking could increase the walker's blood pressure. In order to investigate this, an experiment is designed that will use a set of college student volunteers. None of the students has engaged in regular exercise prior to the experiment, and all of the students are willing to walk every day during the course of the experiment. By random assignment it is determined for each participant whether the person will carry in each hand weights of 2 pounds, 4 pounds, 6 pounds, or no weights at all, and also whether the person will walk 2 miles each day or 4 miles each day. The number of treatments used in this experiment is

(A) 1 (B) 2 (C) 4 (D) 6 (E) 8

Answer

3. In which of the following hypothesis tests could a t-distribution be the most appropriate distribution to use for calculation of the p-value?

(A) A test for a population mean where the population standard deviation is known
(B) A test for a population mean where the population standard deviation is unknown
(C) A test for a population proportion
(D) A goodness of fit test
(E) A test for independence of two categorical variables

Answer

4. Three brothers are arguing as to which of them did best in their end-of-year math exams. Robert, who is in the 10th grade, got an 84. Justin, who is in the 8th grade, got a 93. Bryan, who is in the 7th grade, got an 81. In order to sort out the argument, their mother suggests they ask their teachers what the overall means and standard deviations were for each of the exams. These quantities, along with the brothers' scores, are given in the table below.

	Score	Exam Mean	Exam Standard Deviation
Robert	84	77.1	7.3
Justin	93	82.0	7.8
Bryan	81	73.2	6.9

Using this information, the brothers decide which of them did best, which did second best, and which did third best, relative to their fellow students. Which of the following is the correct list (with the brother who did best relative to his fellow students first in the list)?

(A) Robert, Justin, Bryan
(B) Robert, Bryan, Justin
(C) Justin, Robert, Bryan
(D) Justin, Bryan, Robert
(E) Bryan, Robert, Justin

Answer

5. A large two-year college has found that the number of years completed by the population of students attending the college has the distribution given in the table below.

Number of Years Completed	0	1	2
Proportion of Students	0.08	0.22	0.70

(This tells us that 8% of students attending the college do not complete their first year, 22% complete their first year but not their second year, and the remaining 70% complete the entire two-year program.)

It can be calculated, using the information in the table, that the mean number of years completed for students attending this college is 1.62. Which of the following is the standard deviation of the number of years completed by students attending this college?

(A) $\sqrt{(0-1.62)^2+(1-1.62)^2+(2-1.62)^2}$

(B) $\sqrt{(0-1.62)^2(0.08)+(1-1.62)^2(0.22)+(2-1.62)^2(0.70)}$

(C) $\sqrt{\dfrac{(0-1.62)^2+(1-1.62)^2+(2-1.62)^2}{2}}$

(D) $\sqrt{\dfrac{(0-1.62)^2+(1-1.62)^2+(2-1.62)^2}{3}}$

(E) $\sqrt{\dfrac{(0-1.62)^2(0.08)+(1-1.62)^2(0.22)+(2-1.62)^2(0.70)}{3}}$

Answer

6. A researcher wishes to find out whether ducks of a particular breed tend to lay more eggs in the summer than in the winter. The researcher selects a random sample of ducks of this breed, and, over the course of a year, notes for each duck in the sample the number of eggs laid in the summer and the number of eggs laid in the winter. Which of the following would be a suitable test for analyzing the results of this study?

(A) One-sample z-test for a mean
(B) Two-sample t-test for means
(C) Paired t-test
(D) One-proportion z-test
(E) Two-proportion z-test

Answer

7. A large population has mean μ and standard deviation σ. A random sample of size n will be taken from the population. The Central Limit Theorem tells us that

(A) the mean of the sampling distribution of the sample mean is μ
(B) the standard deviation of the sampling distribution of the sample mean is $\frac{\sigma}{\sqrt{n}}$
(C) if n is large, the sampling distribution of the sample mean is approximately normal
(D) since the population is large, the sampling distribution of the sample mean must be approximately normal
(E) the standard deviation of the sampling distribution of the sample mean is greater than the standard deviation of the population

Answer

8. In a pilot study, a random sample of 98 adults is selected from a large town, and it is found that 33 of these 98 people consume vegetables three or more times per day. A one-proportion z-test is used to analyze this result. According to the test, does this sample result provide convincing evidence at a 5 percent level of significance that more than 25 percent of the adults in the town consume vegetables three or more times per day?

(A) Yes, since the test statistic is less than the critical value.
(B) Yes, since the test statistic is greater than the critical value.
(C) No, since the test statistic is less than the critical value.
(D) No, since the test statistic is greater than the critical value.
(E) The question cannot be answered, since the sample was not large enough for a one-proportion z-test to be used.

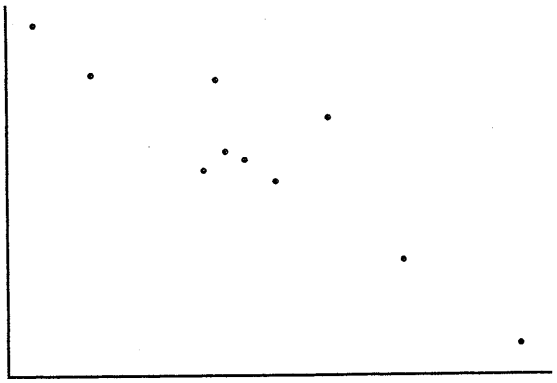
Answer

9. For two events A and B, let $P(A \mid B)$ denote the conditional probability that A occurs given that B occurs, and let $P(B \mid A)$ denote the conditional probability that B occurs given that A occurs. If $P(A \text{ and } B) \neq 0$, and $P(A \mid B) = P(B \mid A)$, which of the following must be true?

(A) A and B are independent.
(B) A and B are mutually exclusive.
(C) A and B have equal probabilities.
(D) The sum of the probabilities of A and B is 1.
(E) $P(A \mid B)$ and $P(B \mid A)$ are both equal to 1.

Answer

10.



Which of the following could be the value of the correlation coefficient for the data set represented by the scatterplot above?

(A) −0.91 (B) −0.34 (C) 0.03 (D) 0.34 (E) 0.91

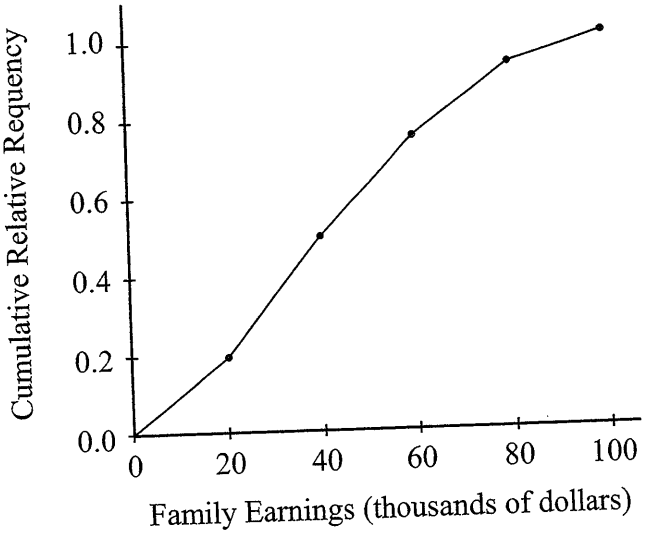
Answer

11. Following an oil spill, a particular region of the ocean is being tested for the level of a chemical called naphthalene. It is considered fact that fish from the region will be safe to eat if, and only if, the mean naphthalene level in the region is less than 3.3 parts per billion. A set of water specimens will be randomly selected from the region and tested, and if the results provide convincing evidence that the mean naphthalene level is less than 3.3, then the sale of fish from the region will be made legal. Which of the following describes a Type I error and its consequences?

(A) The authorities obtain convincing evidence that the mean naphthalene level is less than 3.3, and legalize the sale of fish that are in fact safe for consumption.
(B) The authorities obtain convincing evidence that the mean naphthalene level is less than 3.3, and legalize the sale of fish that are in fact unsafe for consumption.
(C) The authorities fail to obtain convincing evidence that the mean naphthalene level is less than 3.3, and do not legalize the sale of fish from the region when in fact the fish are safe for consumption.
(D) The authorities fail to obtain convincing evidence that the mean naphthalene level is less than 3.3, and do not legalize the sale of fish from the region when in fact the fish are unsafe for consumption.
(E) The definition of a Type I error depends on the actual results of the study in question.

Answer

12. The cumulative relative frequency plot shown below summarizes the family earnings (expressed in US dollars) in a particular country. (The earnings of each family were expressed to the nearest dollar, and the cumulative relative frequencies in the graph indicate the proportions of families earning less than or equal to the amounts shown on the horizontal axis.)



Which of the income brackets below contains the most families?

(A) $0 to $20,000
(B) $20,001 to $40,000
(C) $40,001 to $60,000
(D) $60,001 to $80,000
(E) $80,001 to $100,000

Answer

Questions 13 and 14 refer to the following scenario.

When a person watches a movie, does the video quality (how good the picture and sound are) make any difference as to how high the person rates the content of the movie? Suppose that 90 male college students are shown the same movie, watching in separate locations. By random assignment, 40 of the students are shown the movie in low video quality, with the other 50 students being shown the movie in high video quality. Each student is asked to rate the content of the movie as "Bad," "OK," or "Awesome." The results are shown in the table below.

		Rating of Content Quality			
		"Bad"	"OK"	"Awesome"	Total
Video Quality	Low	15	11	14	40
	High	14	12	24	50
	Total	29	23	38	90

A chi-square test for homogeneity will be used to analyze these results. The expected counts required for the test are shown in the table below.

		Rating of Content Quality			
		"Bad"	"OK"	"Awesome"	Total
Video Quality	Low	12.89	10.22	16.89	40
	High	16.11	12.78	21.11	50
	Total	29	23	38	90

13. Which of the following could be used to calculate the chi-square statistic for this hypothesis test?

(A) $\frac{40 \cdot 29}{90} + \frac{40 \cdot 23}{90} + \frac{40 \cdot 38}{90} + \frac{50 \cdot 29}{90} + \frac{50 \cdot 23}{90} + \frac{50 \cdot 38}{90}$

(B) $\frac{15 - 12.89}{15} + \frac{11 - 10.22}{11} + \frac{14 - 16.89}{14} + \frac{14 - 16.11}{14} + \frac{12 - 12.78}{12} + \frac{24 - 21.11}{24}$

(C) $\frac{|15 - 12.89|}{12.89} + \frac{|11 - 10.22|}{10.22} + \frac{|14 - 16.89|}{16.89} + \frac{|14 - 16.11|}{16.11} + \frac{|12 - 12.78|}{12.78} + \frac{|24 - 21.11|}{21.11}$

(D) $\frac{(15 - 12.89)^2}{15} + \frac{(11 - 10.22)^2}{11} + \frac{(14 - 16.89)^2}{14} + \frac{(14 - 16.11)^2}{14} + \frac{(12 - 12.78)^2}{12} + \frac{(24 - 21.11)^2}{24}$

(E) $\frac{(15 - 12.89)^2}{12.89} + \frac{(11 - 10.22)^2}{10.22} + \frac{(14 - 16.89)^2}{16.89} + \frac{(14 - 16.11)^2}{16.11} + \frac{(12 - 12.78)^2}{12.78} + \frac{(24 - 21.11)^2}{21.11}$

Answer

14. Of the following, which is closest to the true meaning of the p-value for this hypothesis test?

(A) The probability of getting observed counts that are at least as far from the expected counts as was the case in this study given that video quality has no effect on the rating of content quality
(B) The probability of getting observed counts that are at least as far from the expected counts as was the case in this study given that video quality has an effect on the rating of content quality
(C) The probability of getting the observed counts that were obtained in this study given that video quality has an effect on the rating of content quality
(D) The probability that video quality has no effect on the rating of content quality given the results that were obtained in this study
(E) The probability that video quality has an effect on the rating of content quality given the results that were obtained in this study

Answer

15. Tammi's high school consists of 840 students in grades 9-12. Tammi will select a simple random sample of 20 students from the school. Which of the following is NOT true?

(A) This could be achieved by printing the names of all 840 students on identical slips of paper, placing the slips in a large container, picking out 20 slips at random, and including in the sample the 20 students whose names are picked.
(B) This could be achieved by obtaining a list of the 840 students, assigning a distinct random 10-digit number to each student, sorting the list by the size of the random number, and including in the sample the first 20 names on the sorted list.
(C) Any subset of size 20 of the students at the school will have the same probability of being the sample that Tammi selects.
(D) Any student at the school will have the same probability of appearing in the sample.
(E) Tammi's method will ensure that there is an adequate representation of all four grades in the sample.

Answer

16. Which of the following is most likely to be approximately normally distributed?

(A) The scores on a very easy 5-question mental math quiz taken by a large number of students
(B) The scores on a large number of rolls of a cube whose faces are numbered 1 through 6
(C) The number of attempts it took to pass the drivers' test for the population of drivers in New York State
(D) The tail lengths of fully grown males of the common raccoon species
(E) The responses to a survey question given to a large number of people where the possible responses were "yes," "no," and "maybe"

Answer

17. A federation of play-based nursery and elementary schools has found, in the children at its schools, a correlation of 0.65 between time spent playing with brick-based construction toys in nursery school (in minutes per day) and first grade math score (on a scale of 0 to 100). Which of the following is implied by this information?

(A) Playing with brick-based construction toys in nursery school causes an increase in first grade math score.
(B) If the time spent playing with brick-based construction toys had been recorded in hours per day instead of minutes per day, then the correlation would have been 0.65/60.
(C) When time spent playing with brick-based construction toys (in minutes per day) increases by 1, the average increase in first grade math score is 0.65.
(D) If two children are selected from those included in the study, then the one with the greater time spent playing with brick-based construction toys will have the higher first grade math score.
(E) Less than half of the variation in first grade math score can be explained by the regression line of first grade math score on time spent playing with brick-based construction toys in nursery school.

Answer

18. When an experiment is intended to compare two or more treatments, a good design must

(A) include a control group
(B) include some random assignment to treatments
(C) include a group that receives a placebo
(D) involve some sort of blocking
(E) be double blind

Answer

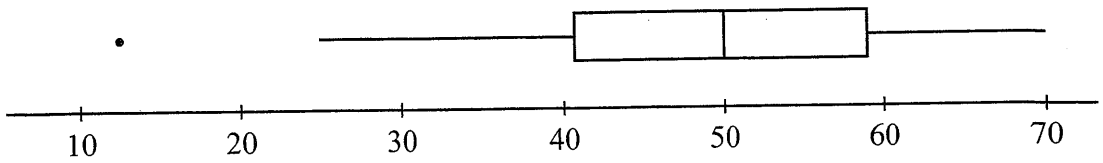
19. Which of the following is true?

I. In a *t*-test for a single population mean, increasing the sample size (while leaving everything else the same) changes the number of degrees of freedom used in the test.
II. In a chi-square test for independence, increasing the sample size (while leaving everything else the same) changes the number of degrees of freedom used in the test.
III. In a *t*-test for the slope of the population regression line, increasing the number of observations (while leaving everything else the same) changes the number of degrees of freedom used in the test.

(A) I only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II and III

Answer

20. The boxplot below represents a data set with one outlier.



If the outlier were to be removed, what would happen to the mean and the standard deviation of the data set?

(A) Both the mean and the standard deviation would remain the same.
(B) Both the mean and the standard deviation would increase.
(C) Both the mean and the standard deviation would decrease.
(D) The mean would increase and the standard deviation would decrease.
(E) The mean would decrease and the standard deviation would increase.

Answer

21. A town has recently moved over to single stream recycling, an approach to waste disposal that recycles a far greater proportion of household refuse than the town's previous approach. Local administrators are interested in determining the degree to which the new approach is being adopted by households in the town. Of the following, which would be likely to produce the most accurate answer to the administrators' question?

(A) Placing researchers at the town's Whole Nutrition food market, and having the researchers select customers of varying genders and ethnicities. The researchers ask the people to state whether they "strongly agree," "agree," "have no opinion," "disagree" or "strongly disagree" with the statement: "My household has substantially changed its approach to refuse disposal as a result of the town's recent change."

(B) Placing researchers at the town's railroad station, bus terminal, mall, and supermarkets, and having the researchers select people of varying genders, ethnicities, and socioeconomic backgrounds. The researchers ask the people to state whether they "strongly agree," "agree," "have no opinion," "disagree" or "strongly disagree" with the statement: "My household has substantially changed its approach to refuse disposal as a result of the town's policies of improving the environment by recycling greater proportions of household waste."

(C) Selecting a large random sample from a complete list of households. Telephone or visit the households selected (repeatedly returning until a response is obtained) asking an adult to state whether he/she "strongly agrees," "agrees," "has no opinion," "disagrees" or "strongly disagrees" with the statement: "My household has substantially changed its approach to refuse disposal as a result of the town's recent change."

(D) Placing leaflets in the mailboxes of all the households in the town asking an adult from the household to respond online to a survey. The online survey asks people to state whether they "strongly agree," "agree," "have no opinion," "disagree" or "strongly disagree" with the statement: "My household has substantially changed its approach to refuse disposal as a result of the town's recent change."

(E) Scheduling a phone-in program on the local radio station that discusses people's feelings and impressions regarding the town's recent change of policy.

Answer

22. Let the proportion of houses in a large city that have mold in their basements be p. When a random sample of n houses is selected from the city, which of the following is the standard deviation of the sampling distribution of $\hat{p}$, the proportion of houses in the sample that have mold in their basements?

(A) $\sqrt{\frac{p(1-p)}{n}}$

(B) $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

(C) $\frac{\hat{p}(1-\hat{p})}{n}$

(D) $\sqrt{np(1-p)}$

(E) $\sqrt{n\hat{p}(1-\hat{p})}$

Answer

23. Jimmy, a very promising 8th grader, is taking an AP Statistics class. The teacher has generated on a computer a large set of numbers that are approximately normally distributed and are being considered as the population. Each student is asked to select a random sample from the population and use the sample to construct a 95% confidence interval for the population mean. Jimmy goes ahead and does this. However, when the teacher announces the true population mean to the class, Jimmy notices that the population mean does not lie within his confidence interval. Which of the following is true?

(A) Jimmy's work could have been correct. Confidence intervals are designed to be narrow, and therefore, for most random samples, the population parameter being estimated will not lie within the interval.

(B) Jimmy's work could have been correct. For about 5% of students doing this exercise correctly the population mean will not lie within the confidence interval calculated.

(C) Jimmy's work could have been correct. It will always be the case that the population mean is not within the confidence interval when the population mean doesn't happen to be in the sample selected.

(D) There must be an error in Jimmy's work. The whole point of confidence intervals is that they should capture the population parameter that is being estimated.

(E) There must be an error in Jimmy's work. He must have calculated an interval that was too narrow, and it was this that caused the interval not to capture the population mean.

Answer

24. Suppose that a detailed study has revealed that for romance novels the number of pages has mean 364 and standard deviation 47, and that for detective novels the number of pages has mean 404 and standard deviation 173. A reader is going to select at random one romance novel, independently select at random one detective novel, and read both books. What is the standard deviation of the total number of pages the person will read?

(A) 14.8 (B) 110.0 (C) 179.3 (D) 220.0 (E) 321.4

Answer

25. The management of a relatively new social networking website named BooglePlus is conducting a pilot study comparing use of its own site with use of a longer established social networking site named FaceList. Some articles published on the Internet give the reader the opportunity to register votes (called "likes") for the article on social networking sites to which the reader belongs. A BooglePlus employee selects from the Internet a random sample of 28 articles where the opportunity is given for registering votes for the article on both BooglePlus and FaceList. Letting x be the number of votes on FaceList and y be the number of votes on the BooglePlus, the slope of the least squares regression line of y on x is found to be 0.0623, with a standard error of 0.0224. Which of the following could be used to compute a 95% confidence interval for the slope of the population regression line of y on x?

(A) $0.0623 \pm (2.056)(0.0224)$
(B) $0.0623 \pm (2.052)(0.0224)$
(C) $0.0623 \pm (2.048)(0.0224)$
(D) $0.0224 \pm (2.056)(0.0623)$
(E) $0.0224 \pm (2.052)(0.0623)$

Answer

26. The housing units on a street have been categorized as to their type ("house" or "apartment") and occupants ("single adult," "couple only," or "with children"). The results are shown in the table below.

		Occupants			
		Single Adult	Couple Only	With Children	Total
Type of Housing Unit	House	5	14	28	47
	Apartment	25	28	20	73
	Total	30	42	48	120

Which of the following is true?

(A) There are more houses than apartments on this street.
(B) More than half of the housing units contain children.
(C) Of the three occupants categories the one with the highest proportion of apartments is "couples only."
(D) Of the two types of housing unit the one with the higher proportion categorized as "couples only" is apartments.
(E) The proportion of houses that contain couples only is smaller than the proportion of apartments that contain children.

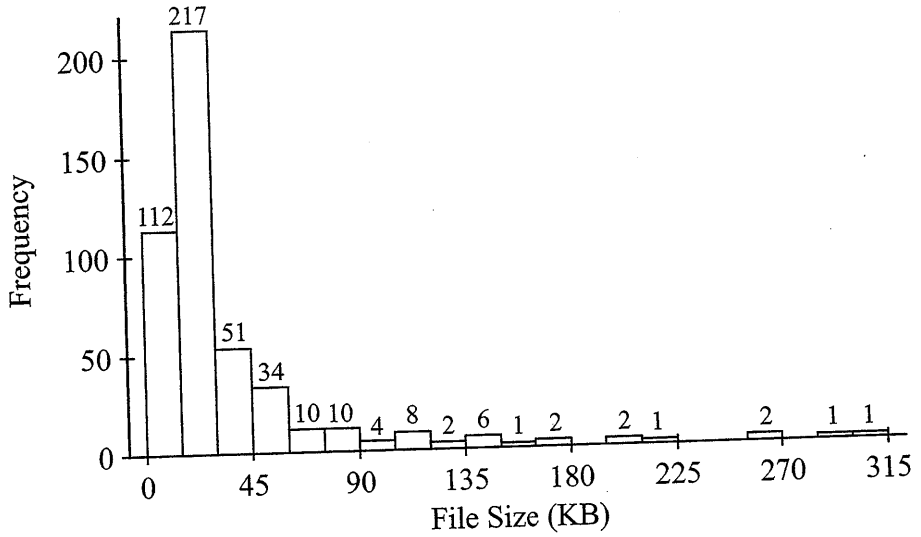
Answer

27. A particular scale of personality was designed so that, in the country where it was formulated, the distribution of scores for the whole country was approximately normal, with mean 100 and standard deviation 15. Which of the following events is most likely?

(A) The mean personality score for a random sample of 9 people from the country is between 95 and 105
(B) The mean personality score for a random sample of 9 people from the country is between 90 and 110
(C) The mean personality score for a random sample of 16 people from the country is between 98 and 102
(D) The mean personality score for a random sample of 16 people from the country is between 95 and 105
(E) The mean personality score for a random sample of 16 people from the country is between 90 and 110

Answer

28. A computer contains on its hard drive 464 spreadsheet files. The sizes of these files, in kilobytes (KB), are summarized in the histogram below.



Which of the following is true about the distribution of file sizes?

(A) The median is greater than 30.
(B) The range is greater than 320.
(C) The first quartile is less than 15.
(D) The interquartile range is less than 30.
(E) The distribution is negatively skewed.

Answer

29. A company is comparing the use of two different robots for the detection of contaminants in river water. A random sample of 10 river locations is selected, and Robot 1 is used to locate a particular contaminant at those locations. Then a random sample of 15 further river locations is independently selected, and Robot 2 is used to locate the same contaminant at these locations. The sample means and standard deviations of the times taken (in minutes) to locate the contaminant are given in the table below.

	Sample Size	Mean	Standard Deviation
Robot 1	10	2.886	0.525
Robot 2	15	3.114	0.644

A two-sample t-test will be used to determine whether the results provide convincing evidence of a difference between the population mean times to locate the contaminant. Which of the following correctly calculates the test statistic?

(A) $\dfrac{2.886 - 3.114}{\sqrt{\dfrac{0.525}{10} + \dfrac{0.644}{15}}}$

(B) $\dfrac{2.886 - 3.114}{\sqrt{\dfrac{0.525}{9} + \dfrac{0.644}{14}}}$

(C) $\dfrac{2.886 - 3.114}{\sqrt{\dfrac{(0.525)^2}{10} + \dfrac{(0.644)^2}{15}}}$

(D) $\dfrac{2.886 - 3.114}{\sqrt{\dfrac{(0.525)^2}{9} + \dfrac{(0.644)^2}{14}}}$

(E) $\dfrac{2.886 - 3.114}{\sqrt{\left(\dfrac{0.525 + 0.644}{2}\right)\left(\dfrac{1}{10} + \dfrac{1}{15}\right)}}$

Answer

30. A small class consists of 8 girls and 4 boys. If a team of 4 students is selected at random, what is the probability that all the students on the team are girls?

(A) 0.141 (B) 0.198 (C) 0.255 (D) 0.312 (E) 0.369

Answer

31. A hypothesis test of the null hypothesis H_0 *versus* the alternative hypothesis H_a is performed, using a significance level of α. If the *p*-value for the test is greater than α, which of the following is a correct conclusion to the test?

(A) H_0 is accepted. We have convincing evidence that H_a is false.
(B) H_0 is not rejected. We do not have convincing evidence that H_a is true.
(C) H_0 is not rejected. We have convincing evidence that H_a is false.
(D) H_0 is rejected. We have convincing evidence that H_a is true.
(E) H_0 is rejected. We do not have convincing evidence that H_a is false.

Answer

Questions 32 and 33 refer to the following scenario.

A study using satellites recorded the mass of ice and the number of lightning flashes per minute in thunderstorm cells over a particular region of the country. Prior to analyzing the data, the researchers transformed both variables using logarithms (base 10). They then performed a linear regression of $\log(y)$ on $\log(x)$, where x = ice mass (in kg) and y = number of lightning flashes per minute.

32. The least squares regression line of $\log(y)$ on $\log(x)$ resulting from the regression analysis was

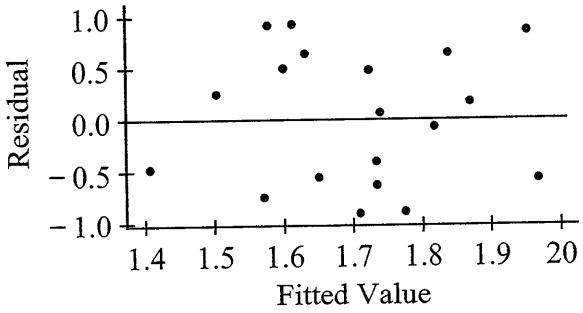
$$\text{predicted value of } \log(y) = -0.285 + 0.2255 \log(x).$$

What does the model predict for the number of lightning flashes per minute when the ice mass is 66,000,000 kg?

(A) 2.1 (B) 5.9 (C) 16.8 (D) 23.8 (E) 30.1

Answer

33. The regression analysis produced the following residual plot.



According to the residual plot, does a linear regression appear to be appropriate for modeling the relationship between the transformed variables?

(A) Yes, because the residual plot shows a random pattern.
(B) Yes, because the vertical spread of points in the residual plot is roughly centered about the horizontal axis that represents a zero residual.
(C) No, because there is no sign of a clear curve in the residual plot.
(D) No, because the best fit line in the residual plot is roughly horizontal.
(E) No, because logarithms base 10 were used, when natural logarithms are more appropriate for use with physical data.

Answer

34. When two cubes, each with faces numbered 1 through 6, are rolled, what is the probability that the total score is 11?

(A) $\frac{1}{36}$ (B) $\frac{1}{24}$ (C) $\frac{1}{18}$ (D) $\frac{1}{12}$ (E) $\frac{1}{6}$

Answer

35. There are large numbers of jobs available in Town A and in Town B. A researcher selects a sample of 97 jobs available in Town A and a sample of 94 jobs available in Town B. The salaries offered for the jobs in the samples are noted, and the results will used to construct a confidence interval for $\mu_A - \mu_B$, where μ_A is the mean salary for all jobs available in Town A and μ_B is the mean salary for all jobs available in Town B. Which of the following is NOT true about construction of the confidence interval, and the interval that is obtained?

(A) It is necessary to assume that the samples are independent and random.
(B) It is necessary to assume that the population distributions are normal.
(C) A t-distribution could be used.
(D) The quantity $\overline{x}_A - \overline{x}_B$ will lie at the center of the interval, where $\overline{x}_A$ and $\overline{x}_B$ are the sample means for Town A and Town B, respectively.
(E) If larger samples had been used, then the confidence interval would probably have been narrower.

Answer

36. A student at a kindergarten-through-12th-grade private school takes a sample of students at the school, and finds, amongst the students in the sample, a positive association between the sizes of their feet and how fast they can run. The student concludes, "All other things being equal, if you want a faster runner you should probably choose someone with large feet." A teacher responds, "That doesn't follow! Kids with bigger feet tend to be older, and older kids tend to be faster runners. For students of the same age, running speed might have nothing to do with foot size!" Which of the following is being considered a confounding variable in this context?

(A) Method of sampling
(B) Method of measuring running speed
(C) Foot size
(D) Age
(E) Running speed

Answer

37. The masses of the berries produced by a particular type of tree can be assumed to be normally distributed with a standard deviation of 72 milligrams. If a berry of this type is selected at random, which of the following represents the probability that the mass of the berry is within 54 milligrams of the mean mass of berries of this type?

(A) $P(z < 0.75)$
(B) $P(z < -0.75)$
(C) $P(z > 0.75)$
(D) $P(z < -0.75) + P(z > 0.75)$
(E) $P(z < 0.75) - P(z < -0.75)$

Answer

38. A simple random sample is selected from a population, and a measurement is taken for each individual in the sample. Using these results, the 95% confidence interval for the population mean is found to be (58.770, 61.428). (The conditions for construction of the interval were checked and verified.) The results from the same sample are now to be used to perform a hypothesis test. If a significance level of $\alpha = 0.05$ is used, which of the following is true?

(A) In a test of $H_0: \mu = 57$ against $H_a: \mu \neq 57$, H_0 would be rejected.
(B) In a test of $H_0: \mu = 57$ against $H_a: \mu < 57$, H_0 would be rejected.
(C) In a test of $H_0: \mu = 59$ against $H_a: \mu \neq 59$, H_0 would be rejected.
(D) In a test of $H_0: \mu = 60$ against $H_a: \mu > 60$, H_0 would be rejected.
(E) In a test of $H_0: \mu = 62$ against $H_a: \mu < 62$, H_0 would not be rejected.

Answer

39. Forty volunteers have been gathered as subjects in an experiment to compare two treatments. These experimental subjects will be randomly assigned to two groups, each of size 20. Random assignment to the groups ensures that

(A) the groups are exactly the same with respect to any variable that might have an effect on a person's response to either of the treatments
(B) any initial differences between the people in the two groups occur completely by chance
(C) the results of the experiment will be the same for the two groups
(D) the participants won't know which treatment they are receiving
(E) the people will not be in the same group as their friends

Answer

40. In which of the following scenarios could a two-proportion z-test be used for the hypothesis test mentioned? (In the scenario or scenarios where the two-proportion z-test is appropriate, you may assume that the conditions for use of the test are met.)

I. A random sample of adults is selected from Neighborhood A and an independent random sample of adults is selected from Neighborhood B. Each adult selected is asked this question: "If you're shopping in a supermarket and the total bill is between \$10 and \$20, do you prefer to pay using cash or some other method?" A hypothesis test is used to compare the proportions of adults in the two neighborhoods responding that they would use cash.

II. A random sample of adults is selected from Neighborhood A and an independent random sample of adults is selected from Neighborhood B. Each adult selected is asked this question: "If you're shopping in a supermarket and the total bill is between \$10 and \$20, do you prefer to pay using cash, a debit card, or a credit card?" A hypothesis test is used to compare the proportions of adults falling into the three categories for the two neighborhoods.

III. The cashiers at a supermarket are given instructions as to how to randomly assign each customer (without the customer knowing) to either Group A or Group B. Customers assigned to Group A will be asked "Would you like to use cash or a card for your payment?" Customers assigned to Group B will be asked "Would you like to use a card or cash for your payment?" A hypothesis test is used to compare the proportions of customers in the two groups who choose to use cash.

(A) I only
(B) II only
(C) I and II
(D) I and III
(E) II and III

Answer

SECTION II
Part A
Questions 1–5
Spend about 65 minutes on this part of the exam.
Percent of Section II grade—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your method as well as on the accuracy and completeness of your results and explanations.

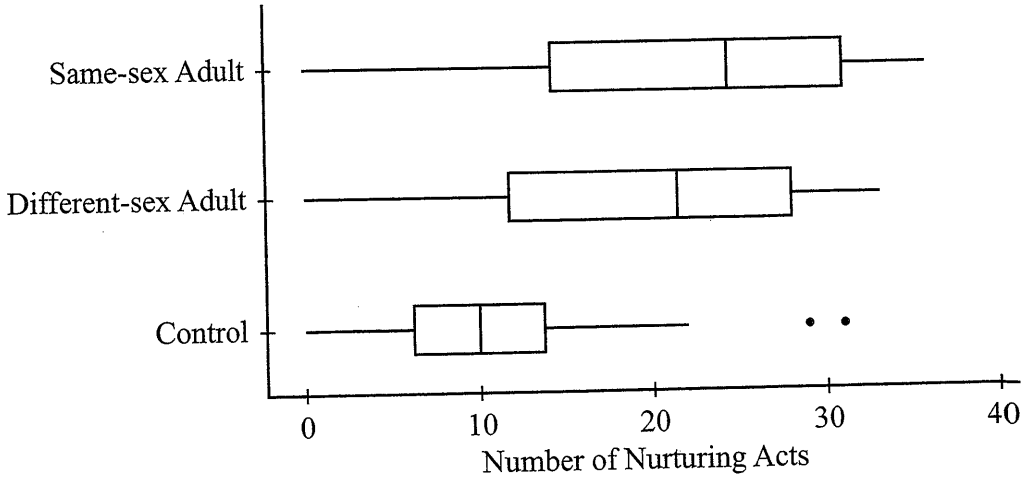
1. An experiment was conducted to compare the influence of an adult of the same sex with that of an adult of the opposite sex in terms of nurturing behavior in children. The experiment used 72 nursery school children aged 3 to 6, and the children were randomly assigned to three groups, with 24 children in each group.

   In the first group, named the "Same-sex adult" group, each child was placed in a room along with some appealing activities such as colored pens and blank coloring books. At the same time, an adult of the same sex as the child was seated in another part of the room with various items including some furry toys. For 10 minutes, the adult showed strong nurturing behavior toward the furry toys while the child played in the other part of the room. Then, for 20 minutes, the child was given unsupervised access to same items as those the adult had used, including the furry toys. During this time the child was observed, and the number of nurturing acts shown towards the furry toys by the child was noted.

   The children in the second group, the "Different-sex adult" group, were given the same treatment as those in the first group, except that the adult model was of the opposite sex to that of the child. Each child's behavior was measured in the same way as in first group.

   Children in the third group, the control group, were given the same treatment, except that the adult involved did not show nurturing behavior towards the toys. (In this group, the sex of the adult was chosen randomly.) Each child's behavior was measured in the same way as in the first two groups.

The results of the experiment are summarized in the boxplots below.



(a) Compare the distributions of the number of nurturing acts among the three groups.

(b) For the "Same-sex adult" group, would the mean most likely have been greater than the median, or less than the median? Explain how you reach your conclusion.

(c) How was the design of this experiment improved by the inclusion of a control group?

2. A consumer organization compared the screen sizes and prices of fourteen global positioning system (GPS) units. The fourteen GPS units used in the study included units with screen sizes as small as 3.5 inches and as large as 7 inches. Prices ranged between \$90 and \$230. Some computer output from a regression analysis of these data is shown below.

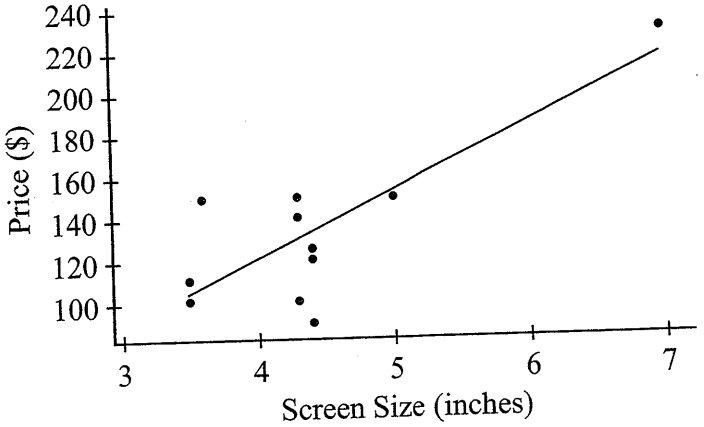
Predictor	Coef	StDev	T	P
Constant	-9.47	29.65	-0.32	0.755
Screen size	32.461	6.777	4.79	0.000
S = 22.3293	R-Sq = 65.7%		R-Sq(adj) = 62.8%	

(a) Using the computer output, write the equation of the least squares regression line that describes the relationship between price and screen size.

(b) Suppose that two GPS units are selected, and the screen sizes of the two units differ by 4 inches. According to the least squares regression line, how much more than the unit with the smaller screen is the one with the larger screen expected to cost?

(c) What does the least squares regression line predict for the price of a GPS unit whose screen size is 6.5 inches?

A scatterplot for this data set, including the least squares regression line, is shown below.



(d) Suppose that the GPS unit with a screen size of 7 inches were to be removed from the data set. Would the line shown in the scatterplot provide the best description of the relationship for the remaining data values? Why, or why not?

3. A study conducted at a large hospital found that 27.8 percent of all patients admitted to the hospital's intensive care unit (ICU) remained in the ICU for less than 24 hours.

   (a) If 8 patients are selected at random from the ICU patients at this hospital, what is the probability that 2 or fewer of them remained in the ICU for less than 24 hours?

   (b) Suppose that 20 patients are selected at random from the ICU patients at this hospital. Calculate the mean and the standard deviation of the number of these patients who remained in the ICU for less than 24 hours.

   (c) Another result of the study was that for elective (non-emergency) admissions to the hospital's ICU, the length of stay in the ICU had a mean of 18.9 hours and a standard deviation of 3.9 hours. Assuming that the length of stay in the ICU for elective admissions is approximately normally distributed, what proportion of elective admissions remained in the ICU for less than 24 hours?

4. The current inventory of a library consists of 198,233 items. The items are categorized as journals, books, DVDs, and other media. The numbers of items falling into the four categories are given in the table below.

Category	Journals	Books	DVDs	Other
Number of Items	124327	56340	10469	7097

(a) Complete the table below showing the proportions of the entire inventory falling into the given categories.

Category	Journals	Books	DVDs	Other
Proportion of Inventory		0.284		

(b) The chief librarian is preparing a report regarding the patterns of use of the library. To assist with this, a coworker compiles data regarding library use during the previous week. The numbers of items used in the four categories during that week are shown in the table below.

Category	Journals	Books	DVDs	Other
Number of Items Used	600	353	58	30

The librarians are willing to treat the uses of the library during that week as a random sample from the set of all uses of the library. Do these results provide convincing evidence that the proportions of all uses that fall into the four categories are different from the proportions of items in those categories (the numbers in the table in part (a))? Provide statistical evidence to support your answer.

5. A student who is writing an article about music for the school newspaper asks another student, Chin-Sun, to estimate the mean length of the mp3 downloads available on a particular web site. Being an AP Statistics student, Chin-Sun decides to make this estimate using a confidence interval. She randomly selects 15 songs from the site, and makes note of their lengths in seconds. Chin-Sun then uses these results to construct a 95% confidence interval for the mean length of all songs on the site. Prior to gathering this sample, Chin-Sun has no knowledge of the lengths of the songs on the site.

   (a) In order to find the critical value to use in the calculation of her confidence interval, should Chin-Sun use the standard normal (z) distribution or a t distribution? Explain your answer.

   (b) What is the meaning of 95% confidence in this context?

   (c) Using the results from her sample, Chin-Sun checks and verifies all the conditions for inference, and correctly calculates the confidence interval for the mean length (in seconds) of all songs on the site to be 242.733 ± 19.209. What was the standard deviation of the song lengths in Chin-Sun's sample?

SECTION II
Part B
Question 6
Spend about 25 minutes on this part of the exam.
Percent of Section II grade—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your method as well as on the accuracy and completeness of your results and explanations.

6. The owner of a small company is planning an economic impact study that will include information about local spending by the company's employees. Spending information is gathered by means of a detailed survey, and so the owner initially plans to select a simple random sample of 8 employees, and require only those 8 people to complete the survey. The company has 32 employees in total.

   (a) Explain how the simple random sample of 8 employees might be selected.

The company's 32 employees are paid at three grade levels: 16 at the "Individual Contributor" level, 12 at the "Professional" level, and 4 at the "Managerial" level. A statistician advising the owner suggests use of a stratified random sample consisting of 4 individual contributors, 3 professionals, and 1 manager.

(b) Why would it be sensible, for the purposes of this study, to stratify by employee grade level?

The economic impact study will include an estimate of the mean local spending for the 32 employees of the company. The statistician wishes to explain further the benefit of using the stratified sampling method described above for estimation of this mean. In order to do this, the statistician creates a list of hypothetical annual local spending values for all 32 employees. These values (in thousands of dollars, rounded to the nearest one-thousand), along with the employees' grade levels, are shown in the table on the next page. (The employees have been sorted according to their hypothetical annual spending values.)

Employee Number	Grade Level	Hypothetical Annual Spending (in thousands of dollars)
5	Individual	9
1	Individual	10
3	Individual	11
2	Individual	13
4	Individual	13
11	Individual	13
14	Individual	13
9	Individual	14
16	Individual	14
6	Individual	15
10	Individual	15
13	Individual	15
23	Professional	15
24	Professional	15
7	Individual	17
8	Individual	17
12	Individual	17
28	Professional	17
15	Individual	18
18	Professional	18
19	Professional	18
20	Professional	18
17	Professional	19
25	Professional	19
26	Professional	19
27	Professional	19
21	Professional	20
22	Professional	20
30	Managerial	20
29	Managerial	22
32	Managerial	22
31	Managerial	26

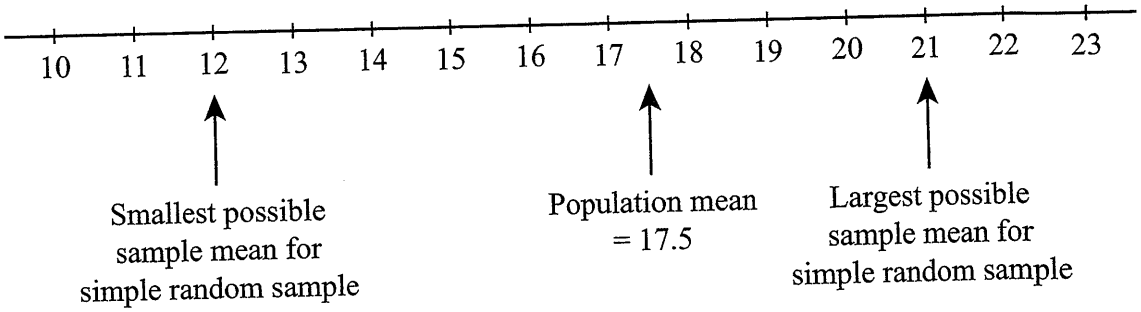
Suppose that a simple random sample of 8 employees is used for the study, and that the hypothetical spending values given in the table are true. Then the smallest possible sample mean for the 8 employees selected is given by

$$\overline{x}_{\min} = \frac{9 + 10 + 11 + 13 + 13 + 13 + 13 + 14}{8} = 12,$$

and the largest possible sample mean is given by

$$\overline{x}_{\max} = \frac{19 + 19 + 20 + 20 + 20 + 22 + 22 + 26}{8} = 21.$$

(c) Suppose, now, that the stratified sampling method described before part (b) is used. Assuming that the spending values given in the table are true, find the smallest and largest possible values of the sample mean, and add your values to the number line given. (The number line also shows the mean hypothetical spending for all employees, which is 17.5.)



(d) The answer to part (c) suggests that one of the two sampling methods produces a sample mean that has a smaller variability than the sample mean produced by the other sampling method. Which of the two sampling methods is this? Explain how you reach your conclusion.

(e) It is known that, over all possible samples, the two sampling methods will both produce sample means that are, on average, equal to the population mean of 17.5. Use your answer to part (d) to explain why, for estimation of the population mean, the stratified random sampling described here would be preferable to simple random sampling.

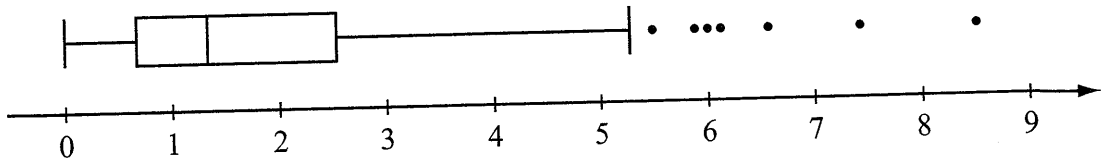
Sample Examination Three

SECTION I

Time—1 hour and 30 minutes
Number of questions—40
Percent of total grade—50

Directions: Solve each of the following problems, using the available space for scratch work. Decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

1. A company has recently given some pay raises. The distribution of the amounts by which the employees' salaries have been increased is illustrated by the boxplot below.



Pay Raise (thousand dollars)

Which of the following best describes the shape of the distribution and the interquartile range (IQR) of the salary increases?

(A) symmetrical; IQR is approximately $700
(B) skewed to the right; IQR is approximately $700
(C) skewed to the left; IQR is approximately $700
(D) skewed to the right; IQR is approximately $2000
(E) skewed to the left; IQR is approximately $2000

Answer

2. A researcher is studying an old edition of an encyclopedia. She wishes to estimate the proportion of the printed matter in the encyclopedia that is diagrams and pictures (as opposed to text). The encyclopedia consists of 30 volumes, and she observes that the proportion of printed matter that is diagrams and pictures is roughly the same in each of the volumes. The researcher randomly selects four of the volumes, and then studies every page in those four volumes. This is an example of which type of sampling?

(A) Cluster
(B) Convenience
(C) Simple random
(D) Stratified random
(E) Systematic

Answer

3. A company produces cloth for use in airplane seats. The company claims that the mean breaking strength μ for specimens of the cloth is 80 pounds of force, but the airlines who buy the cloth are concerned that the cloth might be weaker than that. A group working on behalf of the airlines takes a random sample of specimens of the cloth and finds the breaking strength of each specimen in the sample. What hypotheses should the group use to test the manufacturer's claim?

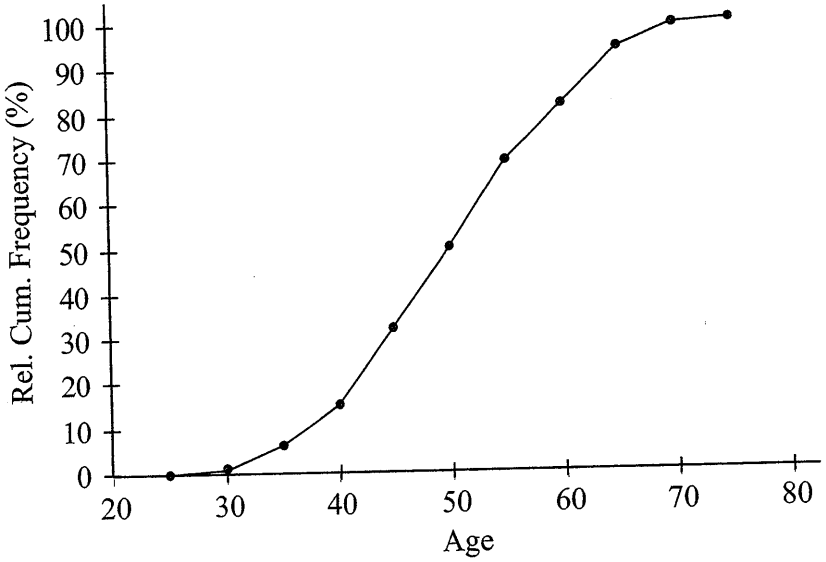
(A) $H_0: \mu = 80, \quad H_a: \mu < 80$
(B) $H_0: \mu = 80, \quad H_a: \mu \neq 80$
(C) $H_0: \mu = 80, \quad H_a: \mu > 80$
(D) $H_0: \mu < 80, \quad H_a: \mu = 80$
(E) $H_0: \mu > 80, \quad H_a: \mu = 80$

Answer

4. In a company, 78% of the employees opt for medical insurance and 42% of the employees opt for life insurance. 82% of the employees opt for at least one of these benefits. What percent of the employees opt for both of these benefits?

(A) 4% (B) 18% (C) 33% (D) 38% (E) 40%

Answer



5. A society has 160 members. A relative cumulative frequency graph of their ages is shown in the figure above. Approximately how many of the society's members are over 43 years old?

(A) 25 (B) 40 (C) 48 (D) 75 (E) 120

Answer

6. A track and field coach wants to find out whether a particular hammer thrower performs better, on average, in the morning or in the afternoon. The coach observes a random sample of the athlete's morning throws and a random sample of the athlete's afternoon throws. Which one of the following significance tests could be used to analyze the results?

(A) One-sample t-test for a mean
(B) Two-sample t-test for means
(C) Paired t-test
(D) One-proportion z-test
(E) Two-proportion z-test

Answer

7. Diana has several children and each of her children has several friends, so she can never be sure how many children will come to dinner. However, over long experience she has worked out that the probability distribution for the number of children who will come to dinner is as shown below.

Number of children	0	1	2	3	4	5	6	7	8	9	10
Probability	0.01	0.04	0.13	0.15	0.16	0.17	0.12	0.09	0.07	0.04	0.02

On any given evening, what is the minimum number of places that she should set at the dinner table for the children in order to be at least 80% sure that all the children can be seated?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

Answer

8. A machine produces metal springs for computer lids. Over a long period of time it has been found that 10% of the springs produced by the machine are defective. After some adjustments to the machine, a random sample of 200 springs is selected and it is found that 16 of the springs in the sample are defective. The appropriate significance test is carried out in order to determine whether the proportion of defective springs has changed. Which of the following is the correct p-value for the test?

(A) $2 \cdot P\left(z < \dfrac{0.08 - 0.1}{\sqrt{\dfrac{(0.1)(0.9)}{200}}}\right)$

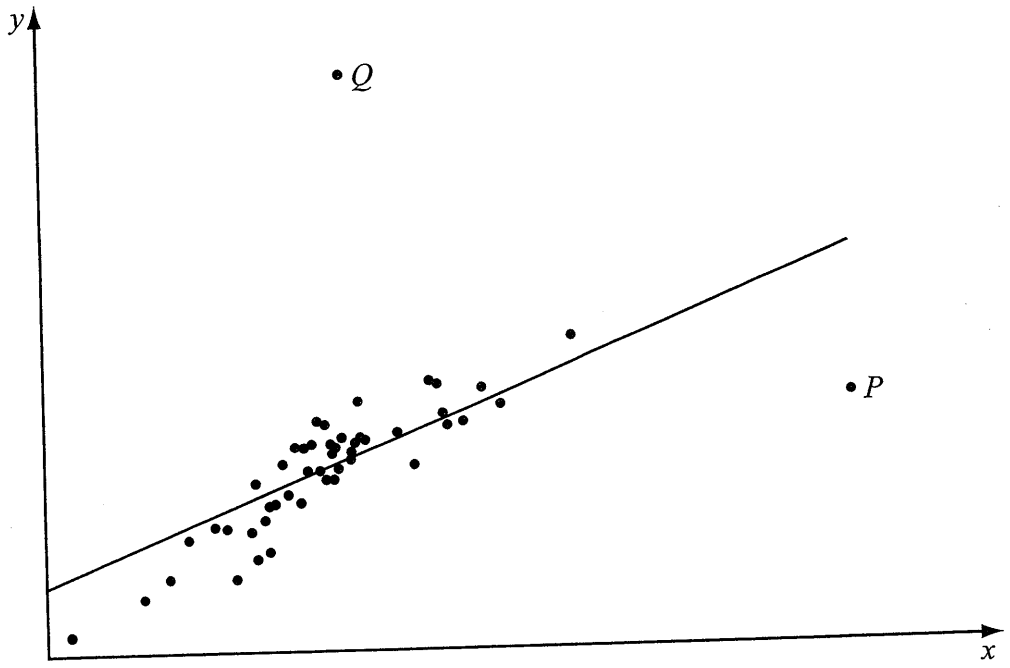
(B) $2 \cdot P\left(z > \dfrac{0.08 - 0.1}{\sqrt{\dfrac{(0.1)(0.9)}{200}}}\right)$

(C) $2 \cdot P\left(z < \dfrac{0.08 - 0.1}{\sqrt{\dfrac{(.08)(.92)}{200}}}\right)$

(D) $2 \cdot P\left(z > \dfrac{0.08 - 0.1}{\sqrt{\dfrac{(.08)(.92)}{200}}}\right)$

(E) $\dfrac{1}{2} \cdot P\left(z < \dfrac{0.08 - 0.1}{\sqrt{\dfrac{(.08)(.92)}{200}}}\right)$

Answer



9. The scatterplot above shows 52 points with the associated least squares regression line for predicting values of y from values of x. One of the two labeled points – either P or Q – will be removed. Which of the following is true?

(A) Removal of the point P would substantially increase the slope of the least squares regression line. Removal of the point Q would have little effect on the slope of the least squares regression line.

(B) Removal of the point P would substantially decrease the slope of the least squares regression line. Removal of the point Q would have little effect on the slope of the least squares regression line.

(C) Removal of the point Q would substantially increase the slope of the least squares regression line. Removal of the point P would have little effect on the slope of the least squares regression line.

(D) Removal of the point Q would substantially decrease the slope of the least squares regression line. Removal of the point P would have little effect on the slope of the least squares regression line.

(E) Removal of the point P would have a substantial effect on the slope of the least squares regression line and removal of the point Q would have a substantial effect on the slope of the least squares regression line.

Answer

10. In a particular country it is known that 40% of the residents have blue eyes, 35% of the residents have brown eyes, and 25% of the residents have green eyes. A student carries out a study to determine whether, in terms of color, the eyes of dolls manufactured in that country are representative of the residents of the country. The student takes a random sample of 40 brands of doll, and finds that 10 of them have blue eyes, 19 of them have brown eyes, and 11 of them have green eyes. She then carries out the appropriate significance test and obtains the p-value for the test. Which of the following is true?

(A) The p-value is between 0 and 0.05.
(B) The p-value is between 0.05 and 0.1.
(C) The p-value is between 0.1 and 0.15.
(D) The p-value is between 0.15 and 0.2.
(E) The p-value is greater than 0.2.

Answer

11. When Joe plays a board game with his four sisters, any one of the five players is equally likely to win. They decide to play the game repeatedly until Joe wins a game, and then they will stop. Assuming that the outcomes of the games are independent, what is the probability that they play at least three games?

(A) 0.128 (B) 0.312 (C) 0.488 (D) 0.512 (E) 0.640

Answer

12. A vending machine delivers varying amounts of coffee. The standard deviation of the amount per serving is known, but the mean amount per serving has recently been adjusted to an unknown value. The person responsible for the machine takes ten servings of coffee from the machine, and is willing to assume that these ten servings form a random sample. She calculates the sample mean serving size to be 10.8 ounces.

She now intends to carry out a z-test about the mean serving size μ for all servings of coffee from this machine. Which of the following pairs of hypotheses will result in the smallest p-value?

(A) $H_0: \mu = 10, \quad H_a: \mu < 10$
(B) $H_0: \mu = 10, \quad H_a: \mu > 10$
(C) $H_0: \mu = 11, \quad H_a: \mu < 11$
(D) $H_0: \mu = 11, \quad H_a: \mu \neq 11$
(E) $H_0: \mu = 11, \quad H_a: \mu > 11$

Answer

13. A student named Russell does a survey concerning the amount of sleep his fellow students are getting. Having taken a sample of students and asked each student the total amount of sleep he/she has had over the past week, he discovers that one of the responses is an outlier. Russell strongly suspects that this response was untrue, but he does not feel able to exclude it from his data set. In order to summarize the center and the spread of the complete set of responses he should quote the

(A) mean and the standard deviation
(B) mean and the interquartile range
(C) mean and the range
(D) median and the interquartile range
(E) median and the range

Answer

14. In a high school there are 638 underclassmen (9th and 10th graders) and 523 upperclassmen (11th and 12th graders). Of the underclassmen 83.1% take the bus to school, and of the upperclassmen 70.9% take the bus to school. If a student is chosen at random from those students who take the bus to school, what is the probability that this student is an underclassman?

(A) 0.457 (B) 0.550 (C) 0.588 (D) 0.671 (E) 0.831

Answer

15. A survey is conducted to compare the proportions of men and women who access their bank statements online. Denoting the population proportions by p_M and p_W, a two-proportion z-test is carried out to test $H_0: p_M = p_W$ against $H_a: p_M > p_W$. The value of the test statistic is found to be $z = 0.784$, and the p-value for the test is found to be 0.216. Which of the following is a correct interpretation of the p-value?

(A) Given the results of the survey, the probability that $p_M > p_w$ is 0.216.
(B) Given the results of the survey, the probability that $p_M = p_W$ is 0.216.
(C) Given that $p_M = p_W$, the probability of getting a value of z at least as large as 0.784 is 0.216.
(D) Given that $p_M > p_W$, the probability of getting a value of z at least as large as 0.784 is 0.216.
(E) Given that $p_M \neq p_W$, the probability of getting a value of z at least as large as 0.784 is 0.216.

Answer

16. A school district currently allows 12th graders at the high school to drive to school. The Board of Education is considering withdrawing this policy, and wishes to determine the opinions of the parents of students in grades K–12 on the issue.

The Board has a list of email addresses covering the parents of most of the students in the district. An email containing the following message is sent to the parents on the list.

> Please read the following statement:
>
> "12th graders should not be allowed to drive to school. The reduction in parking would allow for a substantial expansion in student activities."
>
> Do you strongly agree, agree, disagree, strongly disagree, or have no opinion? Please reply with your response.

After three days, the responses are gathered and are analyzed. Which of the following could NOT be considered a source of bias in this study?

(A) The statement is worded in a way that is likely to influence the reader in a particular direction.
(B) The message is sent to parents of students who are not in the 12th grade.
(C) The list of email addresses does not include the parents of all students in the district.
(D) Some parents will not read the email within the three-day period.
(E) Some parents who read the email will choose not to respond.

Answer

17. For a group of students, the correlation between their heights (in inches) and their weights (in pounds) is 0.332. You are given that 1 inch = 2.54 centimeters and that 1 pound = 0.454 kilogram. If the heights are expressed in centimeters and the weights are expressed in kilograms, what will be the value of the correlation?

A) 0.059 (B) 0.288 (C) 0.332 (D) 0.383 (E) 1.857

Answer

18. A track and field coach has observed two javelin throwers for a long period of time, and now has to select one of them for the team. Which of the following would NOT be a good reason to choose thrower A in preference to B?

(A) The mean for thrower A is greater than the mean for thrower B.
(B) The median for thrower A is greater than the median for thrower B.
(C) The third quartile for thrower A is greater than the third quartile for thrower B.
(D) The maximum for thrower A is greater than the maximum for thrower B.
(E) The distribution of A's throws is positively skewed whereas the distribution of B's throws is roughly symmetrical.

Answer

19. A company is developing a new drug for reducing the symptoms of pollen allergies. They have developed two forms of the drug: A and B. The company wants to find out which form of the drug is most effective and to determine whether the amount to be taken each day should be split into one, two, or three doses. A set of volunteers who suffer from pollen allergies is split into six groups to receive treatments according to the following table.

	1 Dose	2 Doses	3 Doses
Drug A			
Drug B			

How many explanatory variables (factors) are there in this experiment?

(A) 1 (B) 2 (C) 3 (D) 5 (E) 6

Answer

20. An airline observes a random sample of its flights on a particular route. The 95% confidence interval for the mean time (in minutes) for all flights on this route is calculated to be (47.0, 53.0). Which of the following is NOT true?

(A) At the 95% confidence level, the true mean flight time is within 3.0 minutes of the sample mean flight time.
(B) If the true mean flight time were outside the interval (47.0, 53.0) then the sample mean that was found would be very unlikely.
(C) Approximately $2\frac{1}{2}\%$ of flights on this route are longer than 53 minutes.
(D) We are 95% confident that the true mean flight time is between 47.0 and 53.0 minutes.
(E) If many random samples of the same size were taken and the 95% confidence intervals were calculated, then 95% of the confidence intervals would contain the true mean flight time.

Answer

21. A "population" is formed by placing five balls in a bag. The balls are labeled 1, 2, 3, 4, and 5, respectively. The mean of this population is $\mu = 3$. Someone who does not know the contents of the bag will estimate the value of μ by randomly taking a sample of three of the balls (without replacement) and finding either the sample mean or the sample median.

In the meantime, a statistician has listed all the possible samples of size three (sampling without replacement) and has calculated the sample mean and the sample median for each possible sample. The statistician finds that:

- All the possible sample means form a distribution whose mean is 3 and whose standard deviation is 0.577.
- All the possible sample medians form a distribution whose mean is 3 and whose standard deviation is 0.775.

Regarding the choice between using the sample mean and using the sample median for estimating μ, which of the following is true?

(A) Both the sample mean and the sample median are unbiased, but the sample median is preferable as it has the larger standard deviation.
(B) Both the sample mean and the sample median are unbiased, but the sample mean is preferable as it has the smaller standard deviation.
(C) The sample mean is unbiased and the sample median is biased, so the sample mean is preferable.
(D) The sample median is unbiased and the sample mean is biased, so the sample median is preferable.
(E) Both the sample mean and the sample median are biased.

Answer

22. A set of cards cards consists of 12 red cards (numbered 1–12), 12 purple cards (numbered 1–12), 12 green cards (numbered 1–12), and 12 yellow cards (numbered 1–12). One card is going to be picked at random. Let A be the event that the card is green and let B be the event that the card is a 12. Which of the following is true?

(A) The events A and B are independent and mutually exclusive.
(B) The events A and B are independent but not mutually exclusive.
(C) The events A and B are not independent but are mutually exclusive.
(D) The events A and B are not independent and not mutually exclusive.
(E) It is not possible to tell from the information given whether or not the events A and B are mutually exclusive.

Answer

23. A new warm-up procedure has been suggested for use before working out, and it is hoped that the procedure will encourage a greater increase in muscle mass. In order to test this, a study is designed using 40 volunteers who already work out regularly.

The volunteers will be randomly split into two groups, each of size 20. The first group will be taught the warm-up exercises and will be supervised doing the exercises prior to their regular workouts. The second group will merely continue with their regular workouts. At the beginning and at the end of the study, the muscle mass of each of the volunteers will be measured by people who do not know which volunteers were in which group.

Which of the following is NOT the case in the study described?

(A) This study is an experiment.
(B) Randomization is used.
(C) A control group is used.
(D) The study is conducted in a double-blind manner.
(E) There is no blocking involved in the study.

Answer

24. A set of scores has mean 70.3 and a standard deviation 8.8. The scores are now scaled according to the formula $y = 0.7x + 30$, where x is the old score and y is the new score. What is the standard deviation of the new scores?

(A) 4.31 (B) 6.16 (C) 7.36 (D) 36.16 (E) 37.36

Answer

25. In a high school, all of the 11th graders take both math and physics. After the students have taken the midyear exam in both subjects, the physics teachers are considering the results, and have found the value of r^2, the square of the correlation coefficient between the math scores and the physics scores. Which of the following is best answered by consideration of the value of r^2 ?

(A) Whether high physics scores are associated with high math scores
(B) Whether the relationship between physics scores and math scores would be better represented by a curve or a straight line
(C) To what extent the variation in physics scores can be explained by a linear relationship between physics scores and math scores
(D) Whether there is an outlier in the scatterplot of physics scores and math scores
(E) Whether the physics scores are on the whole higher than the math scores

Answer

26. In a random sample of 400 adults, each person stated his or her political preference. The sex (male/female) of each respondent was also noted. The results are shown in the table below.

	Democrat	Republican	Other
Male	94	78	18
Female	88	86	36

If political preference is independent of sex, which of the following is the expected number of respondents who are female and support the Democratic Party?

(A) 40.04 (B) 46.20 (C) 86.45 (D) 95.55 (E) 103.895

Answer

27. Suppose that an observational study has shown that people who regularly consume substantial amounts of olive oil live longer lives, on average, than those who do not. Of the following arguments, which is strongest in explaining why the result of the study does not imply that in order to live longer one should start to regularly consume substantial amounts of olive oil?

(A) Olive oil is high in fat, and it's not a good idea to eat high-fat foods.
(B) There are many other factors contributing to how long you live that were not considered by the study.
(C) If a person is recorded as eating substantial amounts of olive oil and living a long life, we don't know whether the long life was caused by the olive oil eating or, for example, regular exercise.
(D) People who choose to include substantial amounts of olive oil in their diets might well be the sort of people who have healthier lifestyles in general, and a healthy lifestyle leads to a long life.
(E) Olive oil is associated with frying, and frying is unhealthy.

Answer

28. A team of psychologists is studying the behavior of the students in a first grade class. There are 16 girls and 16 boys in the class, and for each student the psychologists record the number of minutes "on task" during a forty minute class. The team wishes to compare the on-task times of the girls with the on-task times of the boys. Which of the following would NOT be a suitable graph for displaying the results?

(A) Parallel dotplots with equal scales
(B) Back-to-back stemplot
(C) Histograms with equal scales
(D) Side-by-side boxplots
(E) Scatterplot with girls' times plotted as x-values and boys' times plotted as y-values

Answer

29. The amount of flour per bag delivered by a machine is known to have a standard deviation of 0.4 ounce. What is the minimum sample size required to estimate the mean amount of flour per bag to within 0.1 ounce with 95% confidence?

(A) 3 (B) 8 (C) 43 (D) 62 (E) 154

Answer

30. In the context of z- and t-tests for the mean using small samples, which of the following is (are) true?

I. The z-test requires the assumption that the population distribution is normal.
II. The t-test requires the assumption that the population distribution is normal.
III. The t-test is used when the population standard deviation is unknown.

(A) I only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

Answer

31. When a large number of a particular type of seed is planted, it is known that 70% of the seeds will germinate. In addition, the germination of any one seed is independent of the germination of any other seed. If 20 of the seeds are planted, what are the mean and the standard deviation of the number of seeds that germinate?

(A) mean = 0.7, standard deviation = 0.102
(B) mean = 0.7, standard deviation = 0.458
(C) mean = 14, standard deviation = 0.102
(D) mean = 14, standard deviation = 2.049
(E) mean = 14, standard deviation = 4.2

Answer

32. A company has a machine that produces cans of coconut milk, and it has been noticed that the amount of coconut milk varies from can to can. The amounts are normally distributed with standard deviation 8 milliliters. The label used on the cans states that each can contains 414 milliliters. The management of the company decides to set the mean μ of the amount of coconut milk per can so that 98% of the cans contain more than 414 milliliters. Of the following, which is the closest to the amount (in milliliters) to which μ should be set?

(A) 395.4 (B) 397.6 (C) 421.8 (D) 430.4 (E) 432.6

Answer

33. A political party wishes to estimate the proportion of voters that support the party in a particular state. The party will poll a random sample of n voters from the state. Which of the following is likely to result in the smallest margin of error?

(A) $n = 400$, confidence level = 95%
(B) $n = 400$, confidence level = 98%
(C) $n = 400$, confidence level = 99%
(D) $n = 500$, confidence level = 95%
(E) $n = 500$, confidence level = 99%

Answer

34. A pharmaceutical company wishes to compare the effectiveness of three drugs, A, B, and C, that are designed to reduce blood pressure. The company believes that the younger a person is, the more likely he is to respond to a drug of this sort. The company intends to design an experiment in which each subject will be instructed to take one of the drugs regularly for a four week period. Each subject's blood pressure will be measured at the beginning and at the end of the four week period.

There are three young men, three middle-aged men, and three elderly men available to take part in this study. Which of the following is the most appropriate method for assigning the treatment groups?

(A) For each man, randomly choose which drug he will be given.
(B) From the whole set of nine men, randomly choose three to receive drug A, three to receive B, and three to receive C.
(C) For the three young men, randomly assign one man to drug A, one man to drug B, and one man to drug C; repeat this process for the middle-aged men and for the elderly men.
(D) Randomly choose one of the drugs, and give that drug to all the young men; randomly choose one of the remaining drugs and give that to all the middle-aged men; and then give the third drug to all the elderly men.
(E) Randomly pick one man from each age group and from these three randomly assign one to drug A, one to drug B, and one to drug C; then pick another man from each age group and do the same thing; then do the same thing for the remaining three men.

Answer

35. A manufacturer of tires has used a particular type of rubber for a long time, and has established over the years that the mean life of the tires is 40,000 miles. However, the company has now changed the type of rubber used and needs to find out whether the mean life of the tires has changed. Having tested a random sample of the tires, a *t*-test for the mean is carried out using H_0: $\mu = 40{,}000$ *versus* H_a: $\mu \neq 40{,}000$. The *t*-value for the test is found to be -1.902 and the *p*-value is found to be 0.063. Using a 5% significance level, which of the following is a correct conclusion for the test?

(A) Since $p > 0.05$ we do not have sufficient evidence to conclude that the mean life of the tires is not equal to 40,000 miles.
(B) Since $p > 0.05$ we do not have sufficient evidence to conclude that the mean life of the tires is less than 40,000 miles.
(C) Since $p > 0.05$ we have sufficient evidence to conclude that the mean life of the tires is equal to 40,000 miles.
(D) Since $p > 0.05$ we have sufficient evidence to conclude that the mean life of the tires is not equal to 40,000 miles.
(E) Since $p > 0.05$ we have sufficient evidence to conclude that the mean life of the tires is greater than 40,000 miles.

Answer

36. A very large population has standard deviation denoted by σ. A random sample of size n will be taken from this population. The quantity $\frac{\sigma}{\sqrt{n}}$ is

(A) the mean of the distribution of the sample standard deviation
(B) the standard deviation of the sampling distribution of the sample mean
(C) the standard deviation of the sample
(D) an estimate of the population standard deviation
(E) an estimate of the sample standard deviation calculated from the population standard deviation

Answer

37. In a test of the null hypothesis H_0: $\mu = 50$ against the alternative hypothesis H_a: $\mu < 50$, with significance level α using sample size n, which of the following is the smallest?

(A) The probability of Type II error when $\mu = 48$, given that $n = 40$ and $\alpha = 0.05$
(B) The probability of Type II error when $\mu = 46$, given that $n = 40$ and $\alpha = 0.05$
(C) The probability of Type II error when $\mu = 48$, given that $n = 40$ and $\alpha = 0.01$
(D) The probability of Type II error when $\mu = 46$, given that $n = 40$ and $\alpha = 0.01$
(E) The probability of Type II error when $\mu = 48$, given that $n = 20$ and $\alpha = 0.05$

Answer

38. Having graded a test, a teacher was interested in the relationship between the amount of time the students studied for the test and the scores they received. She asked the 24 students individually how much they studied, and then compiled a list giving for each student the amount of time studied and the score on the test. The teacher performed a least squares regression analysis. Part of the computer output from that analysis is shown below.

Dependent variable: Score on test

Predictor	Coef	SE Coef	T	P
Constant	69.555194	3.721432	18.69	<.0001
Time	0.2642443	0.109216	2.42	0.0243

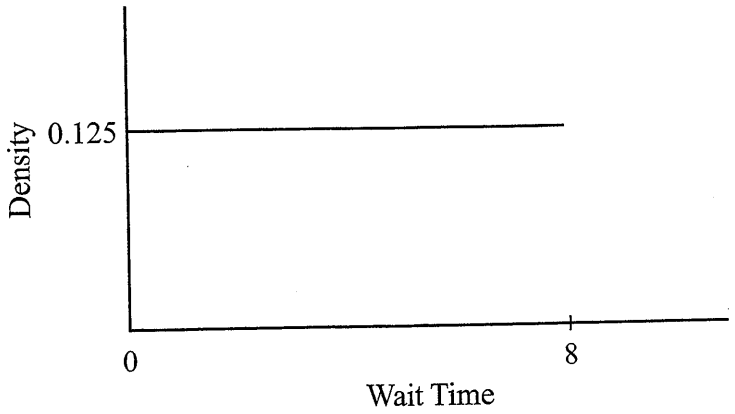
S = 6.3241 R-sq = 21.0% R-sq (adj) = 17.5%

Which of the following is a 99% confidence interval for the slope of the regression line that relates the time spent studying and the score on the test?

(A) $69.555 \pm (2.807)(3.721)$v
(B) $69.555 \pm (2.819)(3.721)$
(C) $69.555 \pm (18.69)(3.721)$
(D) $0.264 \pm (2.807)(0.109)$
(E) $0.264 \pm (2.819)(0.109)$

Answer

39. Every afternoon, Jennifer waits for a subway train. The density curve for the amount of time she has to wait (in minutes) is shown in the diagram below.

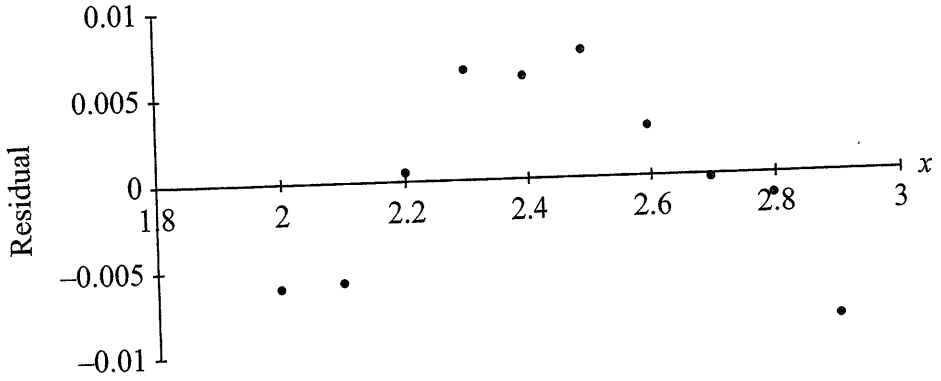


The mean and standard deviation of the wait time are 4 and 2.309, respectively. If a random sample of 40 afternoons is taken, what is the approximate probability that Jennifer's sample mean wait time is less than 5 minutes?

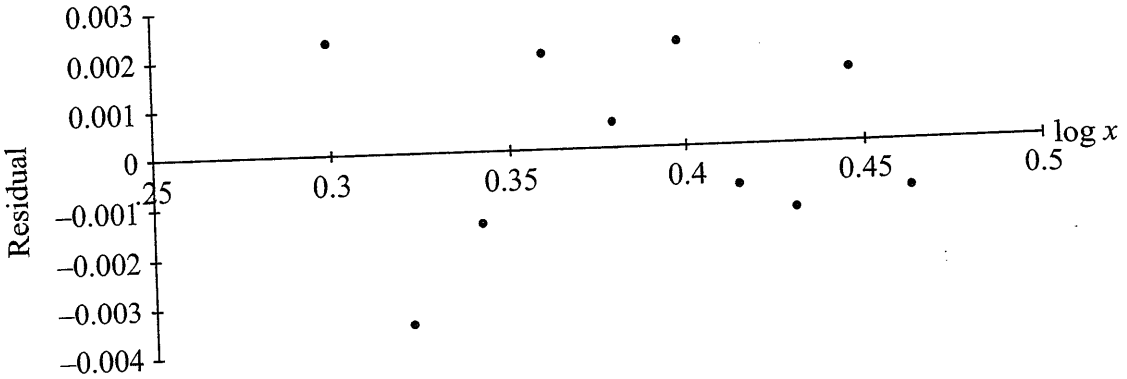
(A) 0.003
(B) 0.332
(C) 0.625
(D) 0.668
(E) 0.997

Answer

40. Two variables, x and y, were measured for a random sample of 10 subjects. In the first of two transformations, log y was plotted (on the vertical axis) against x (on the horizontal axis), a least squares regression was performed on the transformed variables, and the following residual plot was obtained.



In the second transformation, log y was plotted (on the vertical axis) against log x (on the horizontal axis), a least squares regression was performed on the transformed variables, and the following residual plot was obtained.



Which of the following conclusions is best supported by the evidence above?

(A) x and y are related according to an equation of the form $y = ax^p$, where a and p are constants.

(B) x and y are related according to an equation of the form $y = a + x^p$, where a and p are constants.

(C) x and y are related according to an equation of the form $y = a \cdot b^x$, where a and b are constants.

(D) x and y are related according to an equation of the form $y = a + b^x$, where a and b are constants.

(E) x and y are related according to an equation of the form $y = a + b \log x$, where a and b are constants.

Answer

SECTION II
Part A

Questions 1–5

Spend about 65 minutes on this part of the exam.

Percent of Section II grade—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your method as well as on the accuracy and completeness of your results and explanations.

1. For several years, Ellen has been recording music onto audio cassettes. Having accumulated 81 cassettes in this way, she has recently had all of them transferred to digital format, with each cassette going over to one computer file. The sizes of these 81 files in megabytes (MB) are summarized in the table below.

N	MEAN	MEDIAN	STDEV	SE MEAN	MIN	MAX	Q1	Q3
81	544.78	566	167.26	18.58	99	774	463	667.5

(a) Are there any outliers in this data set? Show clearly the method you use to answer this question.

(b) Based on the information given, do you think that the distribution of the file sizes is skewed to the right, skewed to the left, or roughly symmetrical? Explain your answer.

(c) Approximately what percent of the computer files have sizes between 463 and 667.5 megabytes? Explain.

(d) The standard deviation is given as 167.26. Explain how this value summarizes the variability of the file sizes.

2. A driver is interested in buying a new car of a particular type, and she wants to find out how the value of the car is likely to change in its first few years. She randomly selects 20 used cars of this type that are for sale and are at least one year and at most three years old, and notes for each its age (in years) and its price (in dollars). She then uses a computer to fit a least squares regression line to the data. Part of the computer output is shown below.

Dependent variable: Price

Predictor	Coef	SE Coef	T	P
Constant	25844.789	1073.413	24.08	<.0001
Age	-4764.155	526.3235	-9.05	<.0001

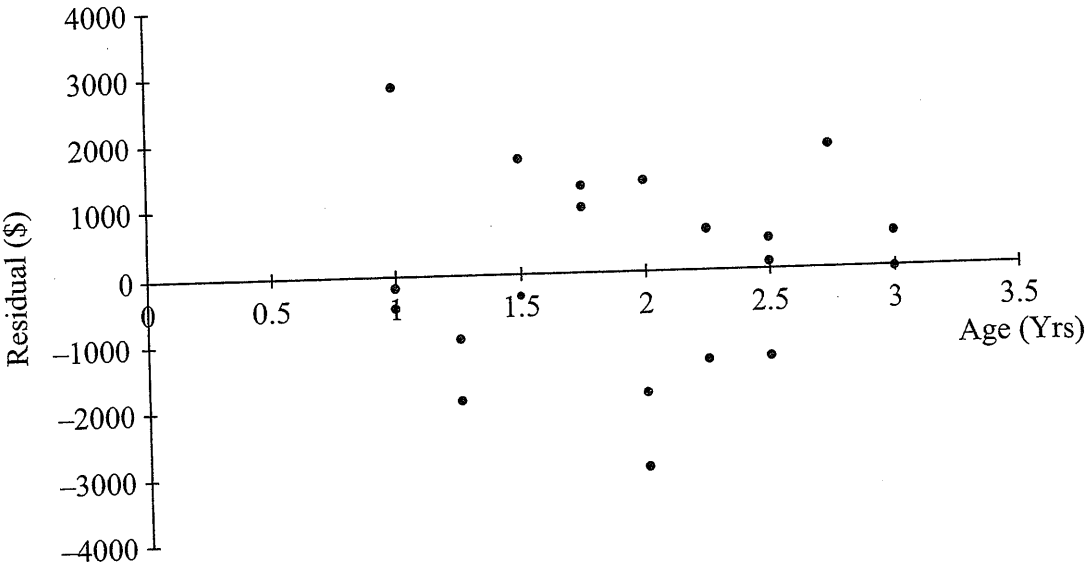
S = 1498.81 R-sq = 82.0% R-sq (adj) = 81.0%

(a) What is the value of the correlation coefficient for age and price? Interpret this correlation.

(b) State the equation of the regression line and interpret its slope in the context of this question.

(c) State and interpret the value of the intercept of the regression line. Can this value be usefully applied to the prices of this type of car?

When the driver used the computer to fit the least squares regression line to the data, the computer also displayed the residual plot shown below.



(d) What does the residual plot tell you about the appropriateness of using the least squares regression line to model the prices of cars of this type that are at least one year and at most three years old? Explain.

3. An investor is trying to decide between two mutual funds, Fund A and Fund B. The investor consults an economist, who estimates that for either of the two funds the amount of money gained in any given month on a $1000 dollar investment is approximately normally distributed, and that the gain or loss in any month can be considered to be independent of the gain or loss in any other month. Moreover, the economist estimates that, if the current economic climate continues, the monthly gains on $1000 in the two funds have the expected values (means) and standard deviations given in the table below.

Gain in dollars on a $1000 investment

	Expected Value	Standard Deviation
Fund A	4	5
Fund B	4	18

Assume throughout this question that the current economic climate continues.

(a) Explain why the investor might choose Fund A over Fund B.

(b) In any given month, what is the probability that Fund A gains money?

(c) What is the probability that Fund A gains money in exactly two of the next four months?

(d) A different investor decides to invest \$8000 in Fund A and \$3000 in Fund B. Assuming that the amounts gained by the two funds are independent, what are the mean and the standard deviation of the amount gained by this investment in the first month?

4. A company is developing a new treatment for gloves worn by gymnasts. The hope is that this treatment will more effectively prevent wear on the gloves than the treatment in use for gloves currently on the market. The company will recruit 50 male gymnasts as volunteers who will wear the gloves as they usually would for six months.

   An employee at the company suggests an experimental design whereby the 50 gymnasts would be randomly assigned to two groups of 25. One group would be given gloves with the new treatment and the other would be given gloves with the current treatment. At the end of the study the wear on the gloves with the new treatment would be compared to the wear on the gloves with the current treatment.

   (a) How would you assign the 50 gymnasts to the two groups of 25 for a completely randomized design?

   (b) Why would the groups be assigned randomly rather than, for example, allowing some of the gymnasts to choose which group they would be in?

The company employs a statistician who suggests a different experimental design. Each of the gymnasts will be given a pair of gloves of which one glove has been treated with the new treatment and the other has been treated with the current treatment. For each gymnast it will be randomly decided whether it is the left glove or the right glove that receives the new treatment.

(c) Explain why this second experimental design is preferable to the first.

5. An experiment was designed to determine whether, in a test of physical endurance, the presence of other participants improved performance. The 32 students in a high school class were randomly assigned to two groups: Group 1 and Group 2. Each student was asked to hold a weight at arm's length in his or her dominant hand for as long as possible. The time (in seconds) for which each student was able to continue to hold the weight in this way was noted. (The same weight was used for each student, and care was taken to ensure that the students' arms were straight, and held in a horizontal position to the side of the body.) Each student in Group 1 performed the task with only the time recorder present. When students in Group 2 performed the task, the other students in Group 2 remained in the room and were allowed to give encouragement to the person performing the task.

   The following results were obtained.

Group 1	180	128	135	207	120	207	159	187	183	83	53	154	72	128	105	227
Group 2	278	126	258	280	225	216	166	138	177	162	301	199	76	390	145	384

   Does the presence of other participants appear to bring about a higher mean time for this task? Give appropriate statistical evidence to support your conclusion.

SECTION II
Part B
Question 6
Spend about 25 minutes on this part of the exam.
Percent of Section II grade—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your method as well as on the accuracy and completeness of your results and explanations.

6. A Board of Education is considering changing the schedule at its two high schools, Central and Northern, so that the school day will start and end one hour later than it does currently. In order to get an idea of student attitudes about the idea, the Board instructs each school to perform a survey on a small sample of its student body.

   (a) The administration of Central High School selects a random sample of 40 students at the school and asks each student in the sample whether or not he/she is in favor of the idea. Twenty-six of the students respond that they are in favor, and the remaining 14 students respond that they are not in favor. Perform a test to determine whether this result provides evidence that a majority of the students at Central High School are in favor of the idea.

The administration of Northern High School decides to include in its survey the possibility of "No Opinion." It designs the following survey question.

"The school day should start and end one hour later."

Disagree (D)_____ No opinion (N)_____ Agree (A) _____

The administration decides that a "D" will score 0, an "N" will score 1, and an "A" will score 2. Having administered the survey to a random sample of 20 students, the administration adds up the scores and finds a total score of 24. An assistant is now given the job of using simulation in order to facilitate a decision as to whether this result reflects support for the idea amongst the student body of Northern High School as a whole. Parts (b), (c), and (d) of this question are concerned with this process.

(b) Suppose that the three responses are favored by equal proportions of students at Northern High School. How would you assign digits in a random number table to simulate the responses to this survey?

(c) Use the random number table given below and your assignment of digits from part (b) to simulate the responses from one sample of 20 students. Show your work clearly on the table, and note the total score for your sample.

4	4	7	3	8	8	5	3	5	1	7	7	6	7	8
6	3	7	6	4	2	5	9	3	9	5	3	8	8	9
6	6	4	8	5	2	5	2	9	9	4	6	9	8	8
1	6	9	0	9	9	0	1	0	4	9	7	4	8	0

The assistant runs the simulation in part (c) 200 times, and obtains the following results.

Total Score	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Number of Runs	1	3	4	3	8	9	16	13	26	11	17	15	28	16	15	6	5	2	0	1	1

(d) On the basis of this set of results (which is based on the assumption that the three responses are favored by equal proportions of the school), do you think that a total score of 24 in a sample of 20 students gives convincing evidence that the student body as a whole is in favor of the idea? Explain carefully the logic behind your answer.